

## Foreword

The *Līlāvātī* of Bhāskarācārya occupies a prominent place among the Sanskrit texts on arithmetic and geometry. With its crisp and precise rules and highly imaginative examples, the *Līlāvātī* became popular throughout India, as is evident from about 600 manuscript copies of this work, written in almost all the scripts in India, which are listed in David Pingree's *The Census of Exact Sciences in India* (vol. A4, Philadelphia 1981). Its popularity is also indicated by the numerous commentaries written on this work.

Outstanding among these commentaries is the *Buddhivilāsinī* of Gaṇeśa Daivajña of Nandigrāma in Konkan. In a *Vasantatilakā* verse at the end of this commentary, Gaṇeśa states that he completed it on *Caitra-śukla-pratipadā*, in the Jovian year Viśvāvasu, 1467 Śaka, which translates to Saturday, 14 March 1545. Like Bhāskarācārya, Gaṇeśa was also a brilliant scholar and poet. He authored several works on astronomy, calendar, astronomical instruments and other related topics. Notable among these works is the *Grahalāghava* which was likewise highly popular.

A conventional commentary explains the meanings of words and their grammatical formation, identifies the metres and the figures of speech, with appropriate citations from lexica and other texts. Since Bhāskara intended that his *Līlāvātī* should be understood even by children without much effort (*bālalīlāvagamya*), Gaṇeśa asserts that in preparing an ordinary commentary on such a lucid text does not involve any great skill; therefore he presents more than a hundred *upapattis* to the rules enunciated in the *Līlāvātī* and thus displays his intellectual ingenuity (*buddhicitram*). Significantly he named his commentary *Buddhivilāsinī*, “that which enlivens or enchants the reader's intellect”. It should be added that the *Buddhivilāsinī* does not contain just *upapattis*; it also performs the normal functions of an ordinary commentary.

*Upapatti*, also called *vāsanā* or *yukti*, is not proof in the Euclidian sense, but rather a justification or rationale for the rules enunciated in the text. Bhāskara himself provides *upapattis* or *vāsanās* in his auto-commentaries entitled *Vāsanābhāṣyas* on his *Bījagaṇita* and *Siddhāntaśiromaṇi*.

These *upapattis* in the *Buddhivilāsinī* are the main focus of Ramakalyani Venkataraman's work *The Buddhivilāsinī Commentary of Gaṇeśa Daivajña on the Līlāvātī of Bhāskara-cārya : A Critical Study*, for which she was awarded the degree of Doctor of Philosophy by the University of Madras in 2018.

This work is divided in nine chapters. The first chapter gives an overview of mathematical texts in Sanskrit and introduces Bhāskara and Gaṇeśa and their works. The next seven chapters discuss and analyse very systematically the *upapattis* presented in the *Buddhivilāsinī* on the successive chapters of the *Līlāvātī*.

In each case, Dr Ramakalyani Venkataraman cites the relevant rule or example from the *Līlāvātī* (either in translation or as paraphrase), reproduces the full text of the *upapatti* on it — after correcting the misprints in the Anandashrama edition —, translates it meticulously into English, explains it in modern notation, oftentimes with suitable diagrams, and discusses it critically. Sometimes she adds insightful remarks or notes on the significance of the concerned *upapatti*. All of this is done in an exemplary manner.

She notes that Gaṇeśa employs different strategies in his *upapattis*. Some are verbal explanations of the procedure of computation, some others employ arithmetical means (*vyakta-rītyā*, *vyakta-kriyayā*), others present algebraic proofs (*avyakta-rītyā*, *avyakta-kriyayā*) with citations from Bhāskara's *Bījagaṇita*, yet others provide geometrical proofs (*kṣetragatopapattyā*), sometimes with appropriate diagrams.

In the *Līlāvātī*, the chapter *Kṣetravyavahāra* dealing with plane geometric figures is the largest chapter. Accordingly, Gaṇeśa's commentary on this chapter is also the largest and the more detailed. Gaṇeśa commences his commentary with a classification of the figures. He observes that there are fourteen types of plane figures, comprising three types of triangles, ten types of quadrilaterals, the last being the circle and bow, which are treated as one type. Other figures like pentagons etc., are treated as combination of two or more triangles.

The final chapter evaluates Gaṇeśa's contribution to mathematics, his style as a commentator and his erudition. The author draws attention to the fact that Gaṇeśa formulates some rules of his own both in verse and prose, viz., rules to determine sine and versine values, the area

of a *cāpakṣetra* (segment of a circle bounded by a chord and bow)), the altitude of *sūcī* triangle and so on and that he adds also some examples of his own.

Gaṇeśa displays his erudition by composing verses with triple or double meaning. The very first verse of the commentary, where he salutes his father and teacher Keśava, has three layers of meaning, all of which he explains meticulously: one pertaining to his father Keśava, another pertaining to Keśava Viṣṇu and a third pertaining to the divine Sun (*praśastā keśā raśmayo yasyāsau keśavaḥ*).

Besides an extensive bibliography of primary and secondary sources, there are six appendices: I. A list of 68 commentaries on the *Līlāvātī*; II. Verses of the *Līlāvātī* for which Gaṇeśa provides an *upapatti*; III. Verses from other Gaṇita texts used in the *Upapattis*, together with English translation; IV. *Bandhas*; V. Figures from Manuscripts; VI. Glossary of Technical Terms.

Of these, Appendix IV deserves special mention. Gaṇeśa displays his virtuosity in versification by composing what are known as *bandhas*, *citra-bandhas* or *citra-kāvya*s (there is no exact English equivalent; in English these are variously called pictorial poetry, figurative poetry, or by the Latin expression *carmena figurata*). These are verses the syllables of which are arranged in different shapes like lotuses, daggers, drums and the like. As Mammaṭa states in his renowned *Kārvyaprakāśa*, such verses are difficult to compose (*kaṣṭam kāvyam etad*). Authors of *Mahākāvya*s like Bhāravi and Māgha composed such verses, but it is very rare to find them in commentaries.

Perhaps the *Buddhivilāsinī* is the only commentary where such *bandhas* occur. There are eight such verses, whose syllables are so arranged that they generate the forms respectively of an eight-petalled lotus (*aṣṭadalapadma-bandha*), parasol (*chatra-bandha*), dagger (*churikā*), conch shell (*śaṅkha-bandha*), bow (*cāpa-bandha*), vessel (*kalāśa-bandha*), trident (*trisūla*) and pearl necklace (*hāra-bandha*). Gaṇeśa generally commences his commentary on each chapter of the *Līlāvātī* with a *bandha*, except at the beginning of the *Rāśi-vyahāra*, *Chāya-vyavahāra*, *Kuṭṭaka* and *Aṅkapāśa*. At the end of the commentary, he gives a verse in *hāra-bandha*. These are not only difficult to compose, but also difficult to understand. Therefore Gaṇeśa explains the meaning of these verses very carefully.

In this Appendix, Dr Ramakalyani Venkataraman presents these eight *citra-bandhas* in elegantly drawn pictorial representations (*nyāsa*), reproduces the verses in readable form and translates them lucidly into English.

In sum, Dr Ramakalyani Venkataraman's critical study of the *Buddhivilāsinī* is a valuable contribution to the historiography of mathematical literature in Sanskrit; in particular, to the study of commentaries on Sanskrit mathematical texts. The study of commentaries on Sanskrit mathematical texts is relatively untouched territory. In 2006 Agathe Keller published a study of Bhāskara I's commentary on the *Āryabhaṭīya*; in 2012 Sita Sundar Ram brought out her study of Kṛṣṇa Daivajña's commentary *Bījapallava* on Bhāskarācārya's *Bījagaṇita*. A short while ago, Ramakalyani Venkataraman has brought out a critical edition of Rāmakṛṣṇa's *Gaṇitāmṛtalaharī* on Bhāskarācārya's *Līlāvātī*, on the basis of eight manuscripts. Now she is enriching the commentarial literature with her excellent study of Gaṇeśa Daivajña's commentary *Buddhivilāsinī* on the *Līlāvātī*.

The credit for sponsoring the study of the two commentaries *Bījapallava* and *Buddhivilāsinī* goes to the Kuppuswami Sastri Research Institute—which bears the name of a great savant—under the able Directorship of Dr V. Kameswari. It is gratifying that D. K. Printworld is bringing out the *Gaṇitāmṛtalaharī* as well as the *Buddhivilāsinī*.

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