

## Historical Note

# Gaṇeśa Daivajña on Multiplication Tables

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### Abstract

Multiplication tables must have been widely prevalent in India since ages, but these are hardly mentioned in Sanskrit texts. Gaṇeśa Daivajña mentions them for the first time in his *Buddhivilāsinī* commentary on Bhāskarācārya's *Līlavatī*. This rare passage from the *Buddhivilāsinī* is discussed in the present article.

**Key words:** *Buddhivilāsinī*, Gaṇeśa Daivajña, Multiplication tables, *Pāṭha*.

## 1. INTRODUCTION

Multiplication tables must be as old as multiplication itself, but we know very little about the nature or form of the multiplication tables in India in earlier times. In my paper on “Some Medieval Arithmetical Tables,” I published some fragments of multiplication tables and tables of squares in Prakrit. There I concluded that multiplication tables are not mentioned in any mathematical text or commentary and that we do not even know the names by which these tables were known in Sanskrit or in other languages before modern times (Sarma 1997, p.193).<sup>1</sup> Subsequently I came across a rare passage which refers to multiplication tables in the Commentary *Buddhivilāsinī* on Bhāskarācārya's *Līlavatī*.

## 2. MULTIPLICATION TABLE IN GAṆEŚA DAIVAJÑA'S *Buddhivilāsinī*

The passage referring to multiplication table in Gaṇeśa Daivajña's *Buddhivilāsinī* commentary (CE1545) on Bhāskara II's *Līlavatī* (verse 14 *ab*) occurs in the context of the latter's first rule on multiplication which reads:

गुणान्त्यमङ्कं गुणकेन हन्याद् उत्सारितेनैवमुपान्तिमादीन् ।

*guṇyāntyam aṅkaṃ guṇakena hanyād  
utsāritenaivam upāntimādīn.*

Multiply the last digit of the multiplicand by the multiplier, [then multiply] the penultimate (*upāntima*) [digit] and so on, by [the multiplier] which is shifted [each time by one place to the right].<sup>2</sup>

After explaining the verbal meaning of this line, Gaṇeśa remarks as follows:<sup>3</sup>

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<sup>1</sup> Sarma, (1997), pp. 191–198, esp. 193: “... there is no clue what their earlier forms must have been in the various Indo-Aryan dialects or in Sanskrit, nor we do we know the names by which they may have been known. There is not a single mention of these in any Sanskrit mathematical text or commentary.”

<sup>2</sup> This is the method of multiplication known as *Kapāṭa-sandhi*; cf. Datta & Singh, (1935–38); second edition, 1962, Part 1, pp. 136–144.

<sup>3</sup> Bhāskarācārya, *Līlavatī*, together with the commentaries *Buddhivilāsinī* by Gaṇeśa Daivajña and *Līlavatīvivarāṇa* by Mahīdhara, ed. Dattātreyā Viṣṇu Āpaṭe, Anandasram Sanskrit Series No. 107, Poona 1937, pp. 14–15.

एवं सर्वस्मिन् गुण्ये गुणिते गुणनफलं स्यात् । रूपगुणम् एतत् ।  
अत्रोपपत्तिः । गुणस्त्वावृत्ति [?]तन्तुष्वित्यभिधानाद् गुणशब्दो  
ऽत्रावृत्तौ वर्तते । अतो द्विरावृत्तो द्विगुणस्त्रिरावृत्तस्त्रिगुण इत्याद्युच्यते ।  
एवं सति

एकेन गुणेनैक एकः, द्वाभ्यां गुण एको द्वौ,  
त्रिभिस्त्रयः, चतुर्भिश्चत्वार इत्यादि.  
एकेन गुणौ द्वौ द्वौ, द्वाभ्यां चत्वारः,  
त्रिभिः षट्, चतुर्भिर्षट्,  
इत्यादीन् एकाद्यङ्कान् एकादिभिर्दशान्तैर्गुणयित्वा सर्वजनैः पठ्यन्ते ।  
तद्यथा-

१	२	३
२	४	६
३	६	९
४	८	१२
५	१०	१५
६	१२	१८
७	१४	२१
८	१६	२४
९	१८	२७
१०	२०	३०

इत्यादि । एवं सुप्रसिद्धपाठेन प्रतिस्थानं गुण्ये गुणिते यथास्थानं  
संयोजिते गुणनफलं स्याद् इत्युपपन्नम् ।

*evam sarvasmin guṇye guṇite guṇanaphalam syāt.  
rūpaguṇam etat. atropapattiḥ. guṇas tv āvṛtti-[-?]-  
tantuṣv<sup>4</sup> ity abhidhānād guṇaśabdo 'trāvṛttau  
vartate. ato dvir āvṛtto dviguṇas trirāvṛttas triguṇa  
ity ādy ucyate. evam saty*

*ekena guṇenaika ekaḥ, dvābhyāṃ guṇa eko dvau,  
tribhis trayāḥ, caturbhiś catvāra ity ādi.  
ekena guṇau dvau dvau, dvābhyāṃ catvāraḥ,  
tribhiḥ ṣaṭ, caturbhir aṣṭau,  
ity ādīn ekādyaṅkān ekādibhir daśāntair guṇayitvā  
sarvajanaīḥ paṭhyante. tad yathā—*

*ity ādi. evam suprasiddha-pāṭhena pratisthānaṃ  
guṇye guṇite yathāsthānaṃ samyojite  
guṇanaphalam syād ity upapannam.*

#### Translation:

Thus when the entire multiplicand is multiplied there will be the product. This is the multiplication [method] of integers (*rūpa*). Here the proof (*upapattiḥ*) is [as follows]. Since the lexicon

(*abhidhāna*) states that the term *guṇa* has the meanings of 'repetition' (*āvṛtti*) ['bow-string' (*vyā*), 'sense organ' (*indriya*), 'secondary' (*amukhya*) and] 'string' (*tantu*), here the term *guṇa* is used in the sense of repetition.

Thus it is said that that which is repeated twice (*dvir āvṛttau*) is two-fold/multiplied by two (*dvi-guṇa*), that which is repeated thrice is three-fold/multiplied by three (*tri-guṇa*), and so on. This being the case,

one multiplied by one [is] one,  
one multiplied by two [is] two,  
[one multiplied] by three [is] three,  
[one multiplied] by four [is] four, and so on.  
two multiplied by one [are] two,  
[two] multiplied by two [are] four,  
[two multiplied] by three [are] six,  
[two multiplied] by four [are] eight.

In this manner all people recite the numbers from one onwards, after multiplying them by [numbers] beginning from one and going up to ten. This is as follows:

1	2	3
2	4	6
3	6	9
4	8	12
5	10	15
6	12	18
7	14	21
8	16	24
9	18	27
10	20	30

and so on. When the multiplicand is multiplied [by the multiplier] at each place according to the well-known recitation (*pāṭha*) [of the multiplication tables] in this way and [the results] added together according to their places, the product is obtained. Thus [the first method of multiplication] has been proved.

Therefore, it appears that at least at the time of Gaṇeśa in the sixteenth century the multiplication tables were known in Sanskrit by the term *pāṭha*, for no intrinsic reason except that these were recited (*paṭhyante*) aloud. This Sanskrit term *pāṭha*

<sup>4</sup> Gaṇeśa's quotation from an unidentified lexicon as printed in the Anandasram edition (*guṇas tv āvṛtti-tantuṣu*) is incomplete. The quotation is from Yādevaprakāśa's *Vaijayantī-kośa* (ed. Gustav Oppert, p. 215, verse 20) where the full quotation reads thus: *guṇas tv āvṛtti-śabdādi-nyendriyāmukya-tantuṣu*. I am highly grateful to Professor K. Ramasubramanian for locating the full quotation.

is clearly the source from which the words for multiplication tables in several NIA or modern north Indian languages are derived, viz. Hindi (*pahāḍā*), Marathi (*pāḍā* / *pāḍhā* / *phāḍā*),<sup>5</sup> Gujarati (*pāḍo*),<sup>6</sup> and Punjabi (*pahārā*).

Bengali, however, has a different term *nāmatā* (supposedly from Sanskrit *nāma-patra*), which appears to be the source for the Assamese *neotā* as well. In Oriya the term is *paṇikiā*.

The situation is entirely different in South Indian languages. Kannada has *maggi* (obviously from Sanskrit *mārga*, paradigm) which was borrowed by Telugu and Konkani. Telugu, at some unidentifiable period, gave up *maggi* in favour of *ekkālu*.<sup>7</sup> The word in Tamil is *perukkal vāyppāṭu* and in Malayalam *guṇanappattigai* (from Sanskrit *guṇana-pattikā*).

In the Sanskrit passage discussed above, the lines *ekena guṇenaika ekah, dvābhyāṃ guṇa eko dvau, tribhis trayah* ... should not be considered a multiplication table in Sanskrit; it is rather a Sanskrit paraphrase of a vernacular table. Children memorize and recite the multiplication tables in a sing-song voice. Therefore the multiplication tables in various vernaculars have an end rhyme, which is missing in the Sanskrit passage. It is certain that in Gaṇeśa Daivajña's time at Konkan,<sup>8</sup> children must have memorized and recited multiplication tables in Old Marathi rather than in Sanskrit. This is implied by Gaṇeśa's statement that multiplication tables are recited by 'all people' (*sarvajanaḥ paṭhyante*).

### 3. CONCLUDING REMARKS

It is highly desirable that multiplication tables in various regional languages (in the oral sing-

song form as they were once taught to children) are collected and documented before they are completely forgotten and lost forever.

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<sup>5</sup> J. T. Molesworth, *Marathi English Dictionary*, sv, at <http://dsal.uchicago.edu/dictionaries/molesworth/> (last accessed in September 2018).

<sup>6</sup> For a fragment of a multiplication table in Old Gujarati which is preserved in Śambhunātha's commentary on the *Pañcaviṃśatikā*, see Takao Hayashi, "Pañcaviṃśatikā in its two Recensions," *IJHS*, 26 (1991): 395–448, esp. 446.

<sup>7</sup> Cf. Sarma, 1987, pp. 163–176.

<sup>8</sup> Cf. Sarma, 2010, pp. 569–574.