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THE *KAṬAPAYĀDI* SYSTEM OF NUMERICAL NOTATION AND ITS SPREAD OUTSIDE KERALA

Sreeramula Rajeswara Sarma

Abstract. — While the study of the transmission of scientific ideas from and to India has its own importance, it is also necessary to examine the transmission of ideas within India, from one region to another, from Sanskrit to regional languages and vice versa. This paper attempts to map the spread of the *Kaṭapayādi* system of numerical notation, widely popular in Kerala, to other parts of India, and shows that this very useful tool of mathematical notation, though well known in northern India, was rarely employed there.

The paper further refutes the contention of Bibhutibhusan Datta and Avadhesh Narayan Singh that there existed four distinct variants of the *Kaṭapayādi* system and shows that there were only two genuine variants of the system, one of these being limited to just one single text of unknown provenance.

Résumé (Le système de notation numérique *Kaṭapayādi* et sa diffusion en dehors du Kérala)

Alors que l'étude de la transmission des idées scientifiques entre l'Inde et le reste du monde a sa propre importance, il est également nécessaire d'examiner la transmission de celles-ci à l'intérieur du pays, entre les régions, de la langue sanskrite vers les langues régionales et vice-versa. Cet article se propose de cartographier la diffusion du système *Kaṭapayādi* de notation numérique, largement répandu au Kérala, aux others parties de l'Inde, et montre que ce très important outil de notation mathématique, alors que bien connu en Inde du Nord, y a été rarement employé.

D'autre part, cet article refute l'affirmation de Bibhutibhusan Datta et d'Avadhesh Narayan Singh sur l'existence de quatre variantes distinctes du

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système *Kaṭapayādi* et montre qu'en réalité il n'y en avait que deux, l'une d'entre elles étant limitée à un seul texte, d'origine inconnue.

INTRODUCTION

Indian astronomers/mathematicians developed two major strategies[1] for expressing large numbers consisting of several digits in the metrical pattern of Sanskrit verses with which they composed their works. In the first method, commonly known as *Bhūtasaṃkhyā* or word numerals, the digits 1 to 9 and zero were expressed by certain significant words.[2] Thus, for example, "moon" stands for 1 as there is only one moon; words like "eye" or "hand" represent 2, because "eyes" or "hands" occur always in pairs. In the second system called *Kaṭapayādi*, the digits are represented by the consonants of the Sanskrit alphabet. For example, 1 is denoted by *k*, *ṭ*, *p*, or *y*. Both the methods operate in the decimal place value system, and in both cases, the enumeration commences with the units' place and proceeds to the next higher powers of ten.

This may be illustrated as follows. In the *Bhūtasaṃkhyā* system, the year 2010 can be expressed as *ākāśa-vasudhā-nabhas-kara*. Here *ākāśa* = sky = 0, *vasudhā* = earth = 1, *nabhas* = sky = 0, *kara* = hand = 2. Since the enumeration begins with the units' place, the order of the digits has to be reversed, and the numerical equivalent of *ākāśa-vasudhā-nabhas-kara* is 2010. With its great wealth of synonyms, Sanskrit is capable of expressing numbers in many ways according to the exigencies of the verse meter; thus 2010 can also be expressed as *ananta-soma-kha-hasta* or *viyac-candra-gagana-nayana* and so on.

In the *Kaṭapayādi* system, 2010 can be denoted by *naṭanara* (*na* = 0, *ṭa* = 1, *na* = 0, *ra* = 2; reading backwards 2010). Here the consonants are chosen in such a way that their combination has some meaning and is not a mere jumble of sounds; *naṭanara* means "actor-man" or "a man who is an actor". Since three or four consonants are available for each digit (there are, however, only two consonants for the zero), and since any vowel can be added to a consonant, a number can be expressed in a variety of ways.

1 In his *Āryabhaṭīya*, Āryabhaṭa I employs an alphabetical notation, which is distinct from the *Bhūtasaṃkhyā* and *Kaṭapayādi*; cf. [Āryabhaṭa I 1976, 1.1, p. 3–5].

2 This system provides symbolic words for a few two-digit numbers as well, such as *dik* (10), *rudra* (11), *tithi* (15), and so on; cf. [Sarma 2003, p. 61–62].

Of these two systems, the *Bhūtasaṃkhyā* is older[3] and was used throughout India and also in South-East Asia, while the latter is essentially a feature of Kerala. In this paper I propose to survey the current state of research on the *Kaṭapayādi* system and discuss some hitherto unknown cases of its spread outside Kerala.

1. THE *KAṬAPAYĀDI* SYSTEM IN KERALA

The *Kaṭapayādi* system is an alphabetical notation where each consonant of the Sanskrit alphabet is given a numerical value.[4] The system is described in an anonymous line thus: *kādi nava, ṭādi nava, pādi pañca, yādy aṣṭau,* "the nine [consonants] starting with *ka*, the nine starting with *ṭa*, the five starting with *pa* and the eight starting with *ya* [respectively denote the numbers 1 to 9]." But it does not say how the zero is to be represented. The *Sadratnamālā* ("Garland of Pure Gems") by Śaṅkaravarman, composed in 1819 AD, gives a comprehensive definition by stating that "*na*, *ña* and the vowels [standing alone denote] zeros; [the consonants] beginning with *ka*, *ṭa*, *pa* and *ya*, [denote severally] the numbers [1 to 9]; in a conjunct [consonant, only] the consonant in the penultimate [place, which is followed by a vowel, denotes a] number; and a consonant not attached to a vowel should not be considered [as a number]."[5] This may be graphically shown in the following table:

1	2	3	4	5	6	7	8	9	0
ka	*kha*	*ga*	*gha*	*ṅa*	*ca*	*cha*	*ja*	*jha*	*ña*
ṭa	*ṭha*	*ḍa*	*ḍha*	*ṇa*	*ta*	*tha*	*da*	*dha*	*na*
pa	*pha*	*ba*	*bha*	*ma*					
ya	*ra*	*la*	*va*	*śa*	*ṣa*	*sa*	*ha*		unattached vowels

3 Symbolic words representing numbers are used sporadically in the *Śrauta-sūtras* and *Vedāṅga-jyotiṣa*. Pingala's *Chandaḥsūtra*, which is generally placed in the second century BC, is the earliest text where these are employed rather extensively, with about a hundred occurrences; cf. [Sarma 2009b].

4 The important literature on the *Kaṭapayādi* system is the following, arranged in chronological order: [Warren 1825, p. 335], [Whish 1827], [Jacquet 1835, p. 122–130], [Ojha 1971, p. 123], [Fleet 1911], [Fleet 1912], [Datta & Singh 1962, Part 1, p. 69–72], [Raja 1963], [Sarma 1972, p. 6–8], [Subbarayappa & Sarma 1985, p. 47–48], [Sarma 2003], [Yano 2006, p. 150–153], [Plofker 2009, p. 75–77].

5 [Śaṅkaravarman 2001, 3.3, p. 22]:
nañāv acaś ca śūnyāni saṃkhyāḥ kaṭapayādayaḥ |
miśre tūpāntahal saṃkhyā na ca cintyo hal asvaraḥ ||
Though the *Kaṭapayādi* system was followed extensively for a very long time in Kerala, it is strange that there are no definitions earlier than this one in the *Sadratnamālā* of the nineteenth century! Perhaps it was thought unnecessary to define it as it was universally known.

Though neither of the definitions expressly states it, the numerals represented in this system are read from the right to the left.[6] This is a neat and elegant method of expressing long numbers in astronomical texts, more so because the numerical phrases, apart from the numerical value they represent, are made up of recognizable words with some meaning. An oft-quoted example is that of Melpatūr Nārāyaṇa Bhaṭṭatiri, a great Sanskrit poet of Kerala, closing his devotional poem *Nārāyaṇīyam* with the expression *āyur-ārogya-saukhyam*.[7] On the one hand it is a benediction for longevity (*āyur*), health (*ārogya*) and happiness (*saukhya*), for himself and for the readers of his work; on the other it is a chronogram indicating the date of completion of the work, viz. 17,12,210 civil days from the beginning of the present Kali era,[8] which translates to 8 December 1586 AD (Gregorian).

What is the antiquity of this system? What is the geographical extent of its use? As with many things Indian, there is no lack of efforts to ascribe "hoary" antiquity to this system, to a period prior to Piṅgala[9] or to the time of the *Mahābhārata*.[10] If that were so, there ought to be abundant examples

6 Following the anonymous maxim *aṅkānāṃ vāmato gatiḥ*, "the digits proceed [from the right] to the left." This is true also of the word notation or the *Bhūtasaṃkhyā* system. On this convention of right-to-left enumeration, see [Sarma 2009b].

7 The final verse of this poem reads as follows [Nārāyaṇa, 100.11]:
ajñātvā te mahattvaṃ yad iha nigaditaṃ viśvanātha kṣamethāḥ
stotraṃ caitat sahasrottaram adhikataraṃ tvatprasādāya bhūyāt |
dvedhā nārāyaṇīyaṃ śrutiṣu ca januṣā stutyatāvarṇanena
sphītaṃ līlāvatārair idam iha kurutām āyurārogyasaukhyam ||
"Forgive, o lord of the universe, whatever I have said here without comprehending your greatness. May this hymn of praise, in [stanzas] more than a thousand, be the source of your grace! [The name of this hymn] *Nārāyaṇīyam* (lit. of / belonging to Nārāyaṇa) [is justified] in two ways: by birth [for it was created by me with the name Nārāyaṇa] and by its praise [of you with the name Nārāyaṇa] in accordance with the scriptures. May this hymn, full of the descriptions of your playful incarnations, confer long life, good health and happiness!"

8 Cf. [Sarma 2003, p. 41]. Here the number given is 17,12,211, obviously a printer's devil.

9 In a highly speculative exercise, [Kak 2000] postulates that the *Kaṭapayādi* (KTPY) notation existed even before the time of Piṅgala who "was the younger brother of Pāṇini" and concludes that his exercise "takes the history of the KTPY mapping more than a thousand years earlier than has been supposed."

10 [Madhavan 1991] is said to have argued that the *Kaṭapayādi* system was used in the *Upadeśasūtra* or *Jaiminisūtra* on astrology by Jamini who is identical with the author of the *Mīmāṃsāsūtra* and therefore "the system probably existed even as early as the fifth century B. C." and even earlier since "The *Mahābharata* is called *Jaya* because *jaya* in *Kaṭapayādi* system means eighteen". [Sarma 2003, p. 43], who refers to this view, adds that it is unacceptable.

of its use from all over India. As will be shown in the following pages, this system of notation was confined only to Kerala, from where it spread sporadically to other parts of India.

The earliest text to employ the *Kaṭapayādi* system is said to be the *Candra-vākyas* ("Moon-sentences") which are attributed to Vararuci. Vararuci is a legendary figure of Kerala who is supposed to have lived in the fourth century AD. But there is no firm evidence to show that he did indeed live in the fourth century AD.[11] According to David Pingree, "the earliest attested epoch of the lunar vākyas is 1184."[12] In his commentary on the *Āryabhaṭīya*, Śūryadeva Yajvan (b. 1191) asserts that Āryabhaṭa I must have known the *Kaṭapayādi* system,[13] implying thereby that the system was already prevalent in the fifth century AD. However, the first positive and datable occurrence is in the *Graha-cāra-nibandhana* which was composed in 683 AD in Kerala by Haridatta, who promulgated through this work the *Parahita* system of astronomy.[14] It cannot be said whether Haridatta himself invented the *Kaṭapayādi* system, but it has been widely

11 [Raja 1963, p. 122], after citing the legend about Vararuci belonging to the fourth century, cautions us, saying that "much credence cannot be given to such stories."

12 [Pingree 1981, p. 47].

13 Commenting on *Āryabhaṭīya*, *Gītikāpāda*, 2 [Āryabhaṭa I 1976, 1.2], where Āryabhaṭa I describes his own system of alphabetical notation, Sūryadeva opines that in the definition of his system of alphabetical notation Āryabhaṭa used the expression *kāt* (i.e. consonants starting from *ka*, and not from *ṭa*, *pa* or *ya*), in order expressly to distinguish his method from the prevailing *Kaṭapayādi* system where consonants severally starting from *ka*, *ṭa*, *pa* and *ya* successively represent the digits from 1 to 9. Cf. [Sūryadeva 1976, p. 10]: *nanu vargākṣarāṇāṃ svarūpavat kakārāditvasyāpi lokasiddhatvāt "kāt" ity etad anarthakam. na. vargākṣarāṇāṃ saṃkhyāpratipādane kaṭapayāditvaṃ nañayoś ca śūnyatvam api prasiddham. tannirāsārthaṃ "kād"-grahaṇam. "kāt" prabhṛty eva vargākṣarāṇāṃ saṃkhyā, na ṭakārāt pakārāc ca prabhṛti. "kāt" prabhṛti sarvāṇi saṃkhyāṃ pratipādayanti, na tu ñakāra-nakārayoś ca śūnyatvam ity arthaḥ.* "[If somebody were to say] 'Now the term *kāt* is redundant because the grouping of the consonants as *vargas* is well known, so too the nature of the consonants *ka* and others', [then our reply is] 'not so'. It is well known that the consonants beginning with *ka*, *ṭa*, *pa*, *ya* in the grouping called *varga* denote the numbers [1 to 9], and *na* and *ña* [denote] zero [in the *Kaṭapayādi* system]. In order to avoid the [possibility that his new system may be confused with the *Kaṭapayādi* system], the term *kāt* has been used by [Āryabhaṭa]. [It means that in Āryabhaṭa's system] the counting of *varga* consonants begins from *ka* only, and not from the letter *ṭa* or *pa*. From *ka* onwards, all [the *varga* consonants] denote numbers [from 1 to 25]; *ña* and *na* do not denote zero."

14 [Datta & Singh 1962, part I, p. 71] say "Its first occurrence known to us is found in *Laghubhāskarīya* of Bhāskara I (522)," and refer to the passage 1.18. However, [Shukla 1976, p. xx-xxii] conclusively demonstrated that Datta & Singh's conjecture of 522 being the date of Bhāskara I and the statement that he used the *Kaṭapayādi* notation are based on a spurious verse found only in one manuscript but not in others.

popular in Kerala at least from his time. Sundararāja (ca. 1475/1500), in his commentary *Laghuprakāśikā* on the *Vākyakaraṇa*, expressly associates the *Kaṭapayādi* system with Haridatta.[15]

The designation of the system as *Kaṭapayādi* (lit. that which begins [severally] with *ka*, *ṭa*, *pa* and *ya*) is attested at least from the ninth century. Kunjunni Raja states that Śaṅkaranārāyaṇa (ca. 825–900) mentions this name in his commentary on the *Laghubhāskarīya* of Bhāskara I.[16] The system is also referred to by its generic name as *Akṣarasaṃjñā* (lit. "alphabetical notation") as will be shown below.

In Malayalam this system is known also as *Paralpperu* where *paral* means "sea shell" and *peru* "name". This expression does not designate the alphabetical notation as such, but merely refers to its application in the astronomical computations performed traditionally in Kerala, using sea shells in this process. Lieutenant Colonel John Warren (1769–1830), [17] an officer in the service of the East India Company and for some time the acting director of the Madras Observatory, was the first European to describe how astronomers performed computations by placing sea shells at appropriate places, while reciting the relevant mnemonic lines with the numerical expressions in the *Kaṭapayādi* notation. The beginning of his description [Warren 1825, p. 334] is cited below. The chapter, which he calls "Fragment IV", has the following heading: *Computation of an Eclipse of the Moon by means of certain memorial and artificial words, and of shells in lieu of figures… By Sami Naden Sashia [=Svāmināthan Śeṣayya], a Kalendar maker residing in Pondicherry.*

> I often read and heard of the singular process by means of which the common Indian Almanac makers computed Eclipses; scoring their quantities with shells, instead of writing them in figures; and dispensing with the use of Tables, by means of certain artificial words, and syllables; which recalled the required

15 Cf. [Sundararāja 1962, p. 37]: *punar apīha … lāghavena vyavahārāya … haridattādibhir aṅgīkṛta-parahitādi-śāstrokta-nyāyena kaṭapayādibhir eva saṃjñābhidhīyate.* "Again here [in this *Vākyakaraṇa*], for the sake of ease in computation, the numbers are denoted by the *Kaṭapayādi* system only, which was adopted (*aṅgīkṛta*) by Haridatta and others and was employed in texts (*śāstra*) like the *Parahita* (i.e., *Grahacāra-nibandhana*) and others."

16 [Raja 1963, p. 129, n. 5], cites [Śaṅkaranārāyaṇa 1949, p. 38]: *kollapuryāṃ viṣuvacchāyāyāḥ pañcadaśasaṃkhyā-saṃpādita-rāśipramāṇāḥ kaṭapayādy-akṣara-baddhāḥ paṭhyante.* "The measures of the equinoctial shadow (*viṣuvacchāyā*) and measures of the zodiac signs, obtained with the number fifteen, in Kollāpurī will be presented in letters beginning with *ka*, *ṭa*, *pa* and *ya*."

17 On Warren, see [Kochhar 1991, p. 98], who likes to narrate that John Warren "was so popular that when his second daughter got married in 1829, the Hindus wanted to pay the expenses of the festivities."

> numbers and Equations to their recollection, and was long desirous to obtain a positive proof of the truth of that report, which I always suspected to be much exaggerated. After a long search for one of these mechanical computers, a person was introduced to me by my venerable friend Abbe *Mottet* (one of the Missionaries of the Institution *de Propaganda Fide* in this part of India), and found the *Sashia* thus introduced to me, competent to my object, for (as I wished) he did not understand a word of the theories of Hindu Astronomy, but was endowed with a retentive memory, which enabled him to arrange very distinctly his operations in his mind, and on the ground.
>
> This person, whose name was *Sami Naden Sashia*, computed before me the Lunar Eclipse which forms the subject of the present Fragment; and after a due examination of his process, I concluded (as I indeed had expected) that the artificial words which were supposed to elicit results, were only designed as vehicles for finding the arguments of the four Vakiam Tables published in this collection, and of some others not included therein, without which it would have been impossible for him to perform his task.[18]

About the geographical extent of its use, Raja [Raja 1963, p. 122] observes as follows: “The *Kaṭapayādi* system is well known only in South India and is most popular in Kerala. [...] It is generally believed to be one of the major contributions of Kerala to Indian mathematics.” That it is employed in Kerala very widely, not only in works on astronomy and mathematics, but in non-scientific works also, not only in Sanskrit writings but in Malayalam as well, is now quite well established [Sarma 2003]. Indeed, in order to facilitate its use in Malayalam texts, the purely Dravidian consonant *ḻ* was also incorporated into the system with the numerical value of 7 [Sarma 1972, p. 6–8]. Even in non-scientific works, the authors preferred to give the date of composition in the form of *Kali-ahargaṇa* (i.e., the number of the civil days elapsed from the beginning of the Kali) in the *Kaṭapayādi* system. Since this number is rather large, the *Kaṭapayādi* system is more convenient than the word numerals; the added advantage being that the poet so chooses his chronogram that it yields, besides the numerical data, some significant or charming meaning as well.

Indeed, it seems that in traditional Kerala, children were taught the *Kaṭapāyadi* system immediately after they had learnt the alphabet. In his memoirs entitled *Ente Smaraṇakal*, Kāṇippayyūr Śaṅkaran Nampūtiripāṭu (1891–1981) narrates as follows: “The next lesson after writing in sand was *akṣarasaṃkhyā* (literally ‘letter number’), which was a useful way of remembering the veritable universe of numerical formulae in astrological calculation, in the days when books and tables either containing or

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18 In Kerala there are still astronomers/astrologers who can compute in this manner with the aid of sea-shells. Their performances deserve to be video-graphed and preserved for posterity.

making unnecessary such formulae, were not published. Each syllable was taken to represent a digit (according to the predominant letter in it) and hence each word, taken as a sequence of syllables, would represent a sequence of digits and thus a number. [...] Hence numerical formulae could be represented in memorized verses, as in some of the astrological verses I learned later on in my primary education."[19]

This system of notation proved to be very useful in Kerala. There it helped the development of a uniform mode of dating events in terms of the *Kali-ahargaṇa*, i.e. the number of civil days from the commencement of the present Kali era at the sunrise on Friday, 18 February 3102 BC (or −3101). *The Kali-ahargaṇa* is akin to, and an early prototype of, the modern method of dating in terms of "Julian days" from the noon of 1 January 4713 BC (−4712),[20] and it is less complicated and more accurate than other calendrical systems.[21] Moreover, the first step in determining the planetary positions on any given day is to compute the number of civil days up to that day from a certain epoch. If the date is given in any other calendrical system, it has to be converted into *ahargaṇa* from the commencement of Kali era or any other epoch. In the case of the *Kali-ahargaṇa* such conversion is not necessary; it can directly be employed in computing the planetary positions.

Likewise, this system facilitated the development of mnemonic tables like the *Candra-vākyas*, which in their turn contributed to the further advances of the Kerala school of mathematics.[22] The *Candra-vākyas* are 248 pithy phrases which state the daily angular motion of the Moon in signs, degrees and minutes. The first phrase *gīr naḥ śreyaḥ*, though it literally means "our welfare [lies in our] speech", gives the parameter 12°03′. The second phrase *dhenavaḥ śrīḥ*, "cows [are] prosperity," 24°09′. While the true moon

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19 Translated and cited by [Wood 1985, p. 37]. It has not been possible for me to find out what sort of mnemonic verses were taught to children so that they became proficient in the *Kaṭapayādi* system.

20 This will be clear by the following example. On 1 January 2010, the *Kali-ahargaṇa* is 1,866,732 and the Julian day number at noon is 2,455,198. The difference between these two numbers, i.e. 588,466, remains constant. In the Islamic world, there is an analogous method, known as *ḥisāb al-jummal*, in which dates are expressed by means of chronograms made up of the *abjad* alpha-numerals [*Encyclopædia of Islam*, vol. 3, 1971, p. 468].

21 In the other systems of dating several variables have be taken into account, such as the different eras, lapsed or current years, solar or lunar months, in the latter case, whether ending in the full-moon (*pūrṇimānta*) or in the new-moon (*amānta*), and so on.

22 On Kerala Mathematics, see, among others, [Joseph 1991, p. 286–294] and [Plofker 2009, p. 217–253, esp. 245–246].

is calculated with these moon-sentences, the true planets are computed with the help of 2075 sentences of the anonymous *Vākyakaraṇa* which was composed in about 1300 [Pingree 1981, p. 48] [Sundararāja 1962].

2. THE SPREAD OF THE *KAṬAPAYĀDI* SYSTEM

From Kerala, the *Kaṭapayādi* system spread to the neighbouring Tamilnadu, where its popularity is fairly well attested. K. V. Sarma who wrote extensively on astronomy and mathematics in Kerala, and also on the history of the *Kaṭapayādi* system, enumerates several cases [Sarma 2003, p. 44]. Sūryadeva Yajvan, who hailed from Gangaikonda-Colapuram, also in Tamilnadu, mentions in one of his works the date of his birth as *vijñānadīptāśayaḥ* (lit. "one whose aspirations are illumined by his knowledge"), numerically 15,68,004 civil days from the commencement of the Kali era.[23] We have also seen from John Warren's testimony that the method of astronomical computations of Kerala was being practiced in Pondicherry in Tamil country still at the beginning of the nineteenth century, using *vākyas* that contain the *Kaṭapayādi* notation.

Outside the texts on mathematics/astronomy, one discipline where the *Kaṭapayādi* system was meaningfully employed in Tamilnadu is the science of music. The musicologist Veṅkaṭamakhin classified the *rāgas* according to their pitch content. In his *Caturdaṇḍi-prakāśikā*, composed in 1660 for the court of Tanjore, he listed all the 72 scales possible in the 12-tone division of the octave. Employing the *Bhūtasaṃkhyā* system, he named the 12 divisions (*cakras*) as *indu* (1), *netra* (2), *agni* (3), *veda* (4), *bāṇa* (5), *ṛtu* (6), *ṛṣi* (7), *vasu* (8), *brahmā* (9), *dik* (10), *rudra* (11) and *āditya* (12). For the individual names of the 72 *rāgas*, he made use of the *Kaṭapayādi* notation in such a way that the first two letters of each name indicate the serial number of the *rāga*. The remaining letters in the name indicate other features of the *rāga*. Thus, e.g. *Kanakāṅgī* shows the serial number 1, *Rūpavatī* 12, *Ṣaṇmukhapriyā* 56, or *Rasikapriyā* 72.[24]

In Karnataka, the *Kaṭapayādi* notation was cultivated by the Jain scholars. In particular, Nemicandra Siddhāntacakravartin (10th century AD) employs it occasionally in his writings, in addition to the ordinary mode

23 Cf. [Sūryadeva 1976, p. xxviii, n.1]. The date translates to 1192 February 4 Tuesday (Gregorian).

24 Cf. [Sarma 2003, p. 40, 43]; [Randel 2003, p. 813]; Wikipedia, s. v. Katapayadi system, accessed on 17 April 2011, contains an interesting chart which shows graphically the 72 *rāgas* along with their attributes.

of expressing numbers. Interestingly enough, Nemicandra uses the letter-numerals rather ambidextrously, that is to say, sometimes in the standard right-to-left mode of enumeration and sometimes in the non-standard left-to-right mode, as shown in the following examples:

2.1. *Ordinary mode of expressing numbers*

Gommaṭasāra, Jīvakāṇḍa, verse 350 [Nemicandra 1974a, p. 426]:

bāruttara-saya-koḍī tesīdī tahaya hoṃti lakkhāṇaṃ |
aṭṭhāvaṇṇa-sahassā paṃceva padāṇi aṃgāṇaṃ ||

"The number of words in the [twelfth] *aṅga* [of the Jain canon] are one hundred and twelve ten-millions (*koḍi*), eighty-three hundred-thousands (*lakkha*), fifty-eight thousands (*sahassa*) and five," i.e. 1,128,358,005.

2.2. *Letter numerals in the standard right-to-left enumeration*

Gommaṭasāra, Jīvakāṇḍa, verse 158 [Nemicandra 1974a, p. 221] [Datta 1930–31, p. 78, n. 4]:

ta-la-lī-na-ma-dhu-ga-vi-ma-laṃ dhū-ma-si-lā-gā-vi-co-ra-bha-ya-me-rū |
ta-ṭa-ha-ri-kha-jha-sā hoṃti hu māṇusa-pajjatta-saṃkhaṃkaṃ ||

Here the number of a class of human beings called *pajjatta* is stated to be
79,228,162,514,264,337,593,543,950,336.

2.3. *Letter numerals in non-standard left-to-right enumeration*

Trilokasāra, verse 98 [Nemicandra 1974b, p. 92] [Datta 1930–31, p. 78, n. 4]:

va-ṭa-la-va-ṇa-ro-ca-gā-na-ga-na-ja-ra-na-gaṃ kā-sa-sa-sa-gha-dha-ma-pa-ra-ka-dha-raṃ |
viguṇa-na-suṇṇa-sahidaṃ pallassa roma-parisaṃkhā ||

"The number of hairs [filling an enormous cosmic space called] *palla* are 413,452,630,308,203,177,749,512,192, followed by twice nine zeros," i.e. 413,452,630,308,203,177,749,512,192,000,000,000,000,000,000

Here it is not expressly stated that the digits are to be read from left to right. But the preceding verse 91 [Nemicandra 1974b, p. 92] states that this number is the product of four factors, viz. *ekkaṭṭhī* (18,446,744,073,709,551,616) × *paṇṇaṭṭhī* (65,536) × *uṇavīsa* (19) ×

aṭṭhārasa (18), and twice nine zeros. Therefore, only the left-to-right enumeration gives the correct product.

The fact that Nemicandra employs the *Kaṭapayādi* notation in both directions suggests that the system is of course known in Karnataka, but not quite well established in the tenth century.

In the Andhra region, however, no scientific or other kind of text appears to have employed this system of notation.

The level of popular usage can be gauged by inscriptions. It is well known that the *Bhūtasaṃkhyā* system of word numerals is widely employed in inscriptions all over India and also in South-East Asia in the chronograms expressing the dates. [Sircar 1965, p. 228–233]. But no comprehensive study has been undertaken so far about the use of the *Kaṭapayādi* system in inscriptions. Gaurishankar Hirachand Ojha lists just five Sanskrit inscriptions that use the *Kaṭapayādi* system, but he does not properly situate the inscriptions as to their time and provenance.[25] An examination of the inscriptions themselves shows that these are rather late records belonging to the 14–16th centuries and emanating from what are today Kerala, Tamilnadu and Andhra Pradesh.

Three of these inscriptions are from Kerala: (i) *rākāloke śakābde*, i.e. Śaka year 1312 = 1389–90 AD; (ii) *śrīmatkolambavarṣe bhavati* denotes the year 644 of the Kolam era, which corresponds to 1468–69 AD; (iii) a*bde kolambavarṣe viśati gavi gurau*, where the chronogram *viśati* has the numerical value of 654, of the Kolam era, which equals 1478–79 AD. [Varmâ 1873].

Two inscriptions are from Tamilnadu. One of these is from the Kāmākṣī Temple of Kāñcīpuram and records the gift of a copper door by Harihara II in *śaktyāloke śakābde*, i.e., Śaka year 1315 (= 1392–93 AD).[26] The other inscription is from Satyamangalam in Vellore district and was issued by Devarāya II. The date is mentioned thus in lines 39–40 [Hultzsch 1894–1895, p. 38]:

tattvāloke śakasyābde krodhisaṃvatsare śubhe |
āṣāḍhāmātithau puṇye somavāravirājite ||

"On Monday, on Amāvāsyā of Āṣāḍha month, in the Jovian year Krodhin, in the Śaka year 1346 (*tattvāloke*)." This corresponds to Monday, 24 July 1424 (Gregorian).

Only one inscription is noticed from the Telugu region. It is incised in Telugu characters on a stone pillar at a temple at Mangalagiri in Guntur

25 Cf. [Ojha 1971, p. 123]; the same is repeated by [Datta & Singh 1962, p. 70–71].

26 It may be noted that in the conjunct consonant *ktyā*, only the final consonant *y* has numerical value.

district and was issued by Kr̥ṣṇadevarāya [Lüders 1900–1901]. It contains several dates expressed in word numerals, occasionally also in ordinary numerals. In two cases, the *Kaṭapayādi* system is used. But the noteworthy feature is that into this Sanskrit inscription a Telugu gloss is inserted which explains that the chronogram is in alphabetical notation. Thus verse 13 which mentions the Śaka year 1437 in word notation is followed by the prose line in Telugu *sāl̥uvāṅka akṣarasaṃjña* | 1437 *śakavarṣālu*, "*sāl̥uvāṅka* is alphabetical notation. [It denotes] 1437 Śaka years."[27] In verse 29, there is again a *Kaṭapayādi* chronogram, followed by a Telugu gloss:

rāghavāya-gaṇite śakavarṣe rāghavāya racitācalapuryām |
vapragopuramayair nava[ha]rmyair gopamantritilakena saparyā ||
rāghavāya 1442 *akṣarasaṃjña*

"In the Śaka year counted by *rā* (2) *gha* (4) *vā* (4) *ya* (1), the illustrious minister Gopa performed the service for Rāghava (god Rāma), in Kondavīdu (*acalapurī*) with new shrines, together with the surrounding walls and high gate towers." [gloss in Telugu]. "*rāghavāya* is the alphabetical notation for 1442".

Does this mean that in the Śaka year 1442 (= 1520 AD), alphabetical notation was still so novel in Andhra region that a gloss in Telugu was thought necessary? Note also that it was called just by the generic term *akṣarasaṃjña* ("alphabetical notation") and not by the specific name *Kaṭapayādi*. Moreover, the last three inscriptions were issued by the kings of Vijayanagara dynasty: Harihara II (Śaka 1315), Devarāya II (Śaka 1346) and Kr̥ṣṇadevarāya (Śaka 1442). D. C. Sircar mentions yet another inscription issued by another king of this dynasty, viz. Sadāśivarāya, in Śaka 1467.[28] It is tempting to think that kings of this dynasty were particularly fond of the *Kaṭapayādi* notation and popularized it throughout their extensive realm which stretched over the modern states Tamilnadu, Karnataka and Andhra Pradesh. One may also conclude that, even though this system may have existed before Āryabhaṭa I in the fifth century AD and even though it was positively attested from the time it was used by Haridatta in 683 AD, it did not become popular enough to be employed in inscriptions until the fourteenth century.

But the inscriptional material gleaned so far is too meager to admit any conclusion. Since the time Ojha collected the five cases of occurrence in

27 [Lüders 1900–1901] notes that here the Dravidian retroflex *ḷ* does not denote 7. Clearly it was treated as identical with ordinary *l* which represents 3.

28 [Sircar 1965, p. 222, n. 1]. The chronogram is *setuvandya*. The inscription is published in *Epigraphia Indica*, vol. 29, p. 71 ff.

1884, no fresh attempt was made during these 126 years to scan the vast corpus of inscriptions for the use of the *Kaṭapayādi* notation, nor, for that matter, were literary works of the regions explored. It is essential to do so in order to clearly map the spread of this system.

There is some evidence to show that this alphabetical notation was known in Orissa. The Orissa State Museum has four manuscripts obviously dealing with this system: No. 2455 *Kaṭapayā* by Vrajasundara Paṭṭanāyaka; No. 2456 *Kaṭapayā, utkalānuvādasametā,* by Kālidāsa Vanamālī; No. 2457 *Kaṭapāyā*; No. 2458 *Kaṭapāyāsūtram* [Misra 1973]. These works appear to be in Sanskrit, and one of them (No. 2456) is said to contain a translation in the Oriya language.

Towards the middle of the tenth century Āryabhaṭa II employed in his astronomical work *Mahāsiddhānta* [1910] [Sarma 1966] a modified version of the *Kaṭapayādi* system [Fleet 1912] [Sarma 1966, part I, p. xx-xxii]. At the very beginning of the work, he defines the new version as follows: "The consonants starting from *ka, ṭa, pa, ya* represent the numerals from one (*rūpa*) [in succession] in the order of the consonants, *ña* and *na* [denoting] the zero." [29] This definition does not fully explain the system; one has to see the examples to understand how this version differs from the standard system. In this version, the consonants have the same value as in the standard system and the vowels—even when standing alone—have no numerical significance. In a conjunct consonant, each member has a numerical value and not just the last consonant as in the standard system. Finally, the digits are read from the left to the right as is the rest of the text, while in the standard *Kaṭapayādi* system the digits are read from the right to the left. Moreover, unlike in the standard system, Āryabhaṭa II makes no attempt to phrase the numerical expressions in such a way that they have some meaning. Consequently his number expressions turn out to be unpronounceable jumbles of sounds.[30] It is not clear why he reversed the general practice of enumerating the digits from the units' place onwards in a right-to-left direction. But the fact that he begins his work with a definition (albeit somewhat deficient) of his new system shows that he is consciously modifying the standard *Kaṭapayādi* system.

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29 [1910, 1.2]:
rūpāt kaṭapayapūrvā varṇā varṇakramād bhavanty aṅkāḥ |
ñnau śūnyaṃ prathamārthe ā chede ai tṛtīyārthe ||

30 Consider, for example, [1910, 2.7], which enumerates the revolutions of the Sun, Moon and Mars in a Kalpa respectively as *gha-ḍa-phe-na-ne-na-na-nū-nī-nā* = 4,320,000,000; *ma-tha-tha-ma-ga-g-la-bha-na-nu-nā* = 57,753,334,000; *kha-kha-jha-ta-jo-gī-pa-nī-ne-nā* = 2,296,831,000.

However, in Chapter 15 on arithmetic and geometry, he employs the word numerals, which he calls "well known symbols" (*prasiddha-saṃjñā*).[31] But the instances where he uses the word numerals are very few, and here he follows the standard practice of enumerating the digits from the units' place onwards. It is not clear either why he makes a special mention that he would use the well known symbols of word numerals in this chapter. He could easily have done with the alpha-numerals as in the rest of the chapters. It is not possible to solve this conundrum with any degree of certainty because, unlike the majority of the Sanskrit writers on astronomy, he does not mention his date, nor does he give any inkling of the region to which he belonged. Because he is mentioned by name by Bhāskara II in his *Siddhāntaśiromaṇi* 1150, he is generally placed in the middle of the tenth century [Pingree 1992]. In any case, no other writer after Āryabhaṭa II followed his modified version of the *Kaṭapayādi* notation.

The only writer outside the south India proper who appears to have employed the standard *Kaṭāpayādi* system in a scientific work is Bhāskara II (b. 1114) of Maharashtra, whose works on astronomy and mathematics attained a near-canonical status [Plofker 2009, p. 182 ff.]. In all his major works, viz. *Līlāvatī, Bījagaṇita, Siddhāntaśiromaṇi* and *Karaṇakutūhala* he makes use of the *Bhūtasaṃkhyā* system, but in his commentary on the *Śiṣyadhīvṛḥddhida* [1981], he uses the *Kaṭapayādi* notation a few times, e.g., *dāsadhirā* 2978 (p. 32), *nidhīdam* 890 (p. 33), *karagaṅgā* 3321 (p. 33) etc. It must remain a matter of speculation why he used the *Kaṭapayādi* system in this commentary and not elsewhere, or why he did not employ the word numerals in this commentary as he did in other works; it is also possible that these cases may be later interpolations.

3. *KAṬAPAYĀDI* SYSTEM IN NORTHERN INDIA

Al-Bīrūnī (973–1048), who spent several years in north-western India and produced an encyclopaedic account of Indian scientific theories and scientific literature, discusses the numerical notation in India. In this connection, he describes only the system of the word numerals, but does not show any awareness of the *Kaṭapayādi* system. In fact, he states expressly

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31 [Āryabhaṭa II 1910, 15.1]:
gaṇite vyavahāre no pāṭījñānād ṛte 'dhikārī syāt |
yasmāt tasmād vakṣye sugamāṃ pāṭīṃ prasiddhasaṃjñābhiḥ ||
"Because one cannot become an expert in mathematics or commerce without the knowledge of arithmetic, I shall teach simple arithmetic with the well known symbols (*prasiddha-saṃjñā*)."

that "[t]he Hindus do not use the letters of their alphabet for numerical notation, as we use the Arabic letters in the order of the Hebrew alphabet."[32] Probably the system did not reach north India in the eleventh century.

However, there is some evidence that the *Kaṭapayādi* system was known in northern India at least from the fifteenth century onwards. It was not used in scientific texts, but only in pseudo-scientific works as a kind of secret code. The textual evidence comes from the works of Rāmacandra Vājapeyin [Sarma 1992], [Sarma 2006], [Pingree 1994, p. 467–478] and his younger brother Harṣa [Devasthali 1943], who flourished in the first half of the fifteenth century at Pātrapuñjangara in Naimiṣāranya, close to modern Sitapur in Uttar Pradesh. Rāmacandra composed many works on exact sciences, such as *Yantraprakāśa* on astronomical instruments, *Śulbavārttika* and *Kuṇḍākṛti* on ritual geometry, but he does not use the *Kaṭapayādi* notation there. He employs it in the *Samarasāra*[33] ("Essence of Warfare"), a slender work of 85 verses which, to judge from the number of extant manuscripts, seems to have been more popular than his scientific works. This text teaches how to draw various diagrams (*cakra*) with different arrangements of the letters of the alphabet. These diagrams are supposed to ensure or forecast victory on the battlefield or in a dispute. Here the *Kaṭapayādi* system is employed not for its numerical values, but to generate entirely different sets of numerical values for the letters of the Sanskrit alphabet. With these new values the names of the two combatants

32 [Al-Bīrūnī 1964, ch. XVI, p. 174]. Subsequently (p. 177–79), Al-Bīrūnī explains how the word numerals are constituted and provides a detailed list of these. On this see also [Chatterjee 1975]. Al-Bīrūnī did not know of the alphabetical notation devised by Āryabhaṭa I. As he states repeatedly [Al-Bīrūnī 1964, i. 370; ii. 16, 33], he had no direct access to Āryabhaṭa's work, his knowledge of Āryabhaṭa I's astronomy being based on Brahmagupta's work which frequently cites the former.

33 There are two printed editions, [Rāmacandra 1982–1983] and [Rāmacandra 1986]. Besides these, I have also used the manuscript [Rāmacandra 1762], which contains the commentary by Bharata.

or disputants are evaluated to forecast who will be victorious.[34] More interesting for the present context is the commentary which is purported to be the joint work of Rāmacandra and his younger brother Bharata.[35] This commentary contains three definitions of the *Kaṭapayādi* system, which are as follows:

kādayo 'ṅkāṣ (9) *ṭādayo 'ṅkāḥ* (9) *pādayaḥ pañca* (5) *kīrtitāḥ* |
yādayo 'ṣṭau (8) *tathā prājñair gaṇakair buddhimattaraiḥ* ||[36]

"The nine [consonants] starting from *ka*, the nine [consonants] starting from *ṭa*, the five [consonants] starting from *pa*, and the eight [consonants] starting from *ya*, are mentioned by the highly intelligent astronomers [as the alphabetical symbols to represent the digits from 1 to 9]."

kaṭapayavargair navanavapañcāṣṭa nañajñāḥ śūnyabodhakāḥ ||

"[The digits 1 to 9 are represented by the] nine [consonants] in sequence starting with *ka*, the nine [consonants] starting with *ṭa*, the five [consonants] starting with *pa*, and the eight [consonants] starting with *ya*, *na*, *ña* and *jña*[37] indicate zero."

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34 For example, verse 6 teaches how to draw the *Jaya-parājaya-cakra* ("Diagram of Victory and Defeat") where each of the cells in the first row is filled with the individual syllables of the phrase *śaṃ-me-gaṅ-gā-ga-ti-ste-da-ha-da-dhi*. The cells in the following rows are filled with the letters of the Sanskrit alphabet in their proper sequence. Now the syllables in the top row *śaṃ-me-gaṅ-gā-ga-ti-ste-da-ha-da-dhi* impart their numerical values according to the *Kaṭapayādi* system to the letters of the alphabet that stand below them in the respective columns. Thus, in the present diagram, the consonants *k*, *ṭ*, *p* and *y* do not have any more the value of 1 as in the standard system, but assume the values of the letters at the top of the respective column. Thus now *k* = 5, *ṭ* = 8, *p* = 8 and *y* = 5.

35 The title of the Jammu edition [Rāmacandra 1982–1983] states that the text is accompanied by a commentary by Bharata (*Bharata-saṃskr̥ta-ṭīkopetam*), but on p. 5, the editor mentions that the commentary is by Rāmacandra and Bharata. This commentary is substantially different from the one in the manuscript used by me. In the Hindi translation, the editor-translator adds one more definition of the *Kaṭapayādi* system which runs thus:
kaṭapayavargabhavair iha piṇḍāntyair akṣarair aṅkāḥ |
niñi (sic!) *ca śūnyaṃ jñeyaṃ tathā svare kevale kathitam* ||
[Subbarayappa & Sarma 1985, p. 48] read the second line as *ne-ñe śūnyaṃ* .... Actually the reading ought to be *na-ñe* ("*na* and *ña*").

36 In the Varanasi edition [Rāmacandra 1986], Rāmajanma Mishra attributes this verse to Rāmacandra himself and adds that this notation was invented for the sake of convenience and secrecy; cf. p. 4–5: *bhaṇitaṃ tathaiva śrī-rāmacandra-ācāryaiḥ: kādayo 'ṅkā ... iti lāghavārthaṃ gopanārthaṃ ca kalpitam.*

37 *Jña* is redundant because in this conjunct consonant only the last member, namely *ña*, has a numerical value which is zero, and this has already been indicated by the previous *ña*.

kādir navāṅkāḥ nava ṭādir aṅkāḥ
pādiḥ śarā yādi bhavanti cāṣṭau |
nañau ca śūnye ca svarāś ca śūnyāḥ ||

"The nine [consonants] starting from *ka*, the nine [consonants] starting from *ṭa*, the five [consonants] starting from *pa*, and the eight [consonants] starting from *ya* [represent the digits 1 to 9]; *na* and *ña* [stand] for zero; and the vowels [represent] zero."

Harṣa, another younger brother of Rāmacandra, composed the *Aṅkayantra-cintāmaṇi*[38] ("Wishing Gem of Magic Squares") to teach how to draw various kinds of magic squares for therapeutic and other purposes.[39] Here verse 5, employing the *Kaṭapayādi* notation, teaches an elegant method of drawing pan-diagonal magic squares of the order four for any desired sum. I reproduce the verse below, after silently correcting the obvious scribal errors:

jñānaṃ śivāpade jñānaṃ jñānaṃ goptṛsurānanam |
iṣṭārdham ūnitaṃ teṣu guptasāraśivāpadaiḥ ||[40]

"[In the 16 cells of the square, write sucessively] 0 (*jñā*), 0 (*naṃ*), 5 (*śi*), 4 (*vā*), 1 (*pa*), 8 (*de*), 0 (*jñā*), 0 (*naṃ*), 0 (*jñā*), 0 (*naṃ*), 3 (*go*), 6 (*ptṛ*), 7 (*su*), 2 (*rā*), 0 (*na*), 0 (*naṃ*). [Then fill those cells having zeros by] half the desired sum reduced [successively] by 3 (*gu*), 6 (*pta*), 7 (*sā*), 2 (*ra*), 5 (*śi*), 4 (*vā*), 1 (*pa*), 8 (*daiḥ*)."

Let the desired sum be $2x$ and its half x. Then the magic square is constructed in the manner shown in Fig. 1 below. In the preceding verse, Harṣa explains which sum would produce what results.[41] The most potent, according to him, is the square with the magic constant 64 which brings

38 I have used the manuscript from the Bombay University Library [Harṣa]; for its description, see [Devasthali 1944, Part II, No. 1719].

39 On the use of magic squares in Sanskrit medical texts, see [Roşu 1987], [Roşu 1986] and [Roşu 1989, p. 127, n. 37] where several extant manuscripts of the *Aṅkayantracintāmaṇi* are listed.

40 See f 3r of the manuscript [Harṣa] mentioned above. In the top margin of this page, there occurs the following gloss: *atha mūlaparibhāṣā. kādi nava 9 ṭādi nava 9 pādi pañca 5 yādy aṣṭau 8.*

41 *nārījīvaśiśuḥ śatena bhuvanaṃ vaśyaṃ catuḥṣaṣṭito*
dvāṣaṣṭer jitavādakaṃ sthitaśubhaṃ dhānyaṃ caturviṃśateḥ |
aṣṭāviṃśatir ītināśana catuḥpañcāśato suprasūḥ
ṣaḍviṃśena rajasvalā gatabhayā dvātriṃśatā yātriṇaḥ ||4||

"By 100 (i.e. by means of a magic square with a constant of 100) the woman's life and the infant [will be saved]; by 64 one has the world under one's control; by 62 the disputant is defeated; 24 will make the fresh crop secure and nourishing; 28 will destroy the seasonal calamities; 55 will help easy delivery; 26 removes the fear (pains) the whole world under one's control. Substituting then x by 32, the magic square for the sum 64 is obtained as shown in Fig. 2.

$x-3$	$x-6$	5	4
1	8	$x-7$	$x-2$
$x-5$	$x-4$	3	6
7	2	$x-1$	$x-8$

Fig. 1

29	26	5	4
1	8	25	30
27	28	3	6
7	2	31	24

Fig. 2

In Fig. 2, the sum of the numbers in any vertical column is 64; so also the sum of any horizontal row and the sum of each diagonal. Furthermore, any shorter diagonal parallel to the main diagonal, together with its complement, will also yield the same sum 64. For example, $27+8+5+24=64$. Therefore, this magic square is called pan-diagonal. In fact, it is much more than a pan-diagonal magic square; here the magic constant is produced also by the sum of any 2×2 square, including those that wrap around the edges of the whole square (e.g. $30+6+1+27=64$), by the sum of the digits in the four corners, by the sum of the digits at the corners of any 3×3 square, and so on [Sarma 2004].

Harṣa's magic square was adopted by some writers in the subsequent centuries. Thus, Raghunandana Bhaṭṭācārya (fl. ca. 1520–1575) [Pingree 1994, p. 341–368] of Bengal, in his *Jyotistattva*, prescribes this magic square with a slight modification by moving the top row to the bottom, for various therapeutic and other purposes. However, he uses the word numerals there and not the alphabetical notation. While Harṣa gives a mechanical rule for creating the square, Raghunandana's rule shows the logic behind the arrangement of the digits [Grierson 1881]. Nīlakaṇṭha Caturdhara, in his commentary on the *Śivatāṇḍavatantra*, which he completed in 1680 at Benares, makes use of this square with an explicit reference to Harṣa. He uses the alphabetical notation, but is also familiar with Raghunandana's rule for filling the cells [Minkowski 2008] [Roşu 1989, p. 129]. There may be similar other uses of the *Kaṭapayādi* notation in the vast pseudo-scientific literature. [42]

Through the contact with Islamic astronomy in the medieval period, astronomers in India became familiar with the alpha-numeric system called *abjad*, which is analogous to the *Kaṭapayādi* system and which was employed in the Islamic world to represent numbers in scientific texts,

of the menstruating woman and 32 [will remove the fears of] the travellers." Though metrically correct, the text of the verse is rather corrupt.

42 See [Goonetilleke 1882]. [Weber 1863, p. 771 ff.] reports that the *Tilaka* commentary on the *Rāmāyaṇa* mentions the number of verses in each canto in the *Kaṭapayādi* system.

in astronomical tables called *zīj-s*, and in the scales engraved on astronomical instruments like astrolabes and celestial globes. As a result of this encounter, Indian astronomers produced several books of astronomical tables in Sanskrit.[43] They also caused the production of astrolabes and celestial globes with legends in Sanskrit. In the books of tables as well as on the astronomical instruments, the numbers are represented by Devanāgarī numerals. Indian astronomers saw clearly that the *Kaṭapayādi* system of notation, though useful in mnemonic verses, does not give any advantage over common numerals in astronomical tables or on astronomical instruments. But two instrument makers decided to imitate the Islamic instruments in all respects in their Sanskrit instruments and therefore labelled the scales with the *Kaṭapayādi* notation. An anonymous Sanskrit astrolabe, now preserved in the Sampurnanand Sanskrit University at Varanasi, displays the *Kaṭapayādi* notation in some of the scales. The astrolabe is not dated; but on technical and stylistic considerations, it can be attributed to the seventeenth century. It was produced somewhere near Delhi, possibly at a place called Vaṃśāvatī-nagara situated on the latitude 28° N. (Fig. 3) [Sarma 1999].

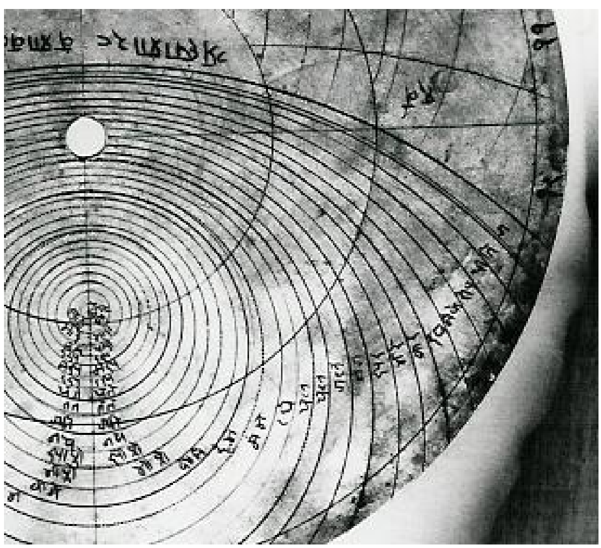

Fig. 3: Plate of the Sanskrit astrolabe

43 Cf. [Plofker 2009, p. 255–278, esp. 274–277].

Bhālūmal of Lahore [Sarma 1994, p. 522–523], who made astrolabes, celestial globes and other instruments in Arabic as well as in Sanskrit, produced in 1839 a Sanskrit celestial globe on which the scales were labelled in the *Kaṭapayādi* notation. This globe is now in a private collection in Milan.[44] In this globe, Bhālumal employs the *Kaṭapayādi* notation on the horizontal ring to represent groups of 6 degrees of arc thus: *ca* (6), *khaya* (12), *jaya* (18), *ghara* (24), *nala* (30), *cala* (36), *khava* (42), *java* (48), *vaśa* (54), *naṣa* (60), *caṣa* (66), *khasa* (72), *jasa* (78), *vaha* (84), *najha* (90). On the globe itself, both the ecliptic and the equator scales are likewise labelled in the *Kaṭapayādi* notation. Here groups of 6° are serially numbered. For economy of space, two consonants are conjoined here, both having numerical value. Thus *pla* (31), *phla* (32), *bla* (33), *bhla* (34), *mla* (35) .... *kva* (41), *khva* (42), *gva* (43), *ghva* (44), *ṇva* (45), *cva* (46), *chva* (47), *jva* (48) ... (See Figure 4).

Fig. 4: Sanskrit celestial globe made by Bhālūmal in 1839

44 The description is based on photographs kindly provided by Professor G. L'E. Turner, Oxford.

4. VARIANTS OF THE *KAṬAPAYĀDI* SYSTEM

In their *History of Hindu Mathematics*, Bibhutibhusan Datta and Avadhesh Narayan Singh declared in 1935 that there were four variants of the *Kaṭapayādi* system [Datta & Singh 1962, part I, p. 69–72], and this has been repeated by several writers since then. It is time that this assertion is reexamined. Datta and Singh mention the following four variants.

(i) The one described in the *Sadratnamālā*, which was quoted at the beginning of this paper. This is the standard system which originated in Kerala and is used there extensively.

(ii) The system followed by Āryabhaṭa II (see p. 49 above).

(iii) "A third variant of this system is found in some Pālī manuscripts from Burma. This is in all respects the same as the first variant except that $s = 5$, $h = 6$, $l' = 7$. The modification in the values of these letters are (sic!) due to the fact that the Pālī alphabet does not contain the Sanskrit *ś* and *ṣ*."[45]

(iv) "A fourth variant of the system was in use in South India, and is known as the Kerala System. This is the same as the first variant with the difference that the left-to-right arrangement of letters, just as in writing numerical figures, is employed."

What Datta and Singh call the third variant is found only in two passages in Pali texts composed in the nineteenth century in Burma. L. D. Barnett drew attention to the relevant expressions in 1907 and J. F. Fleet discussed these in 1911, citing the full passages [Fleet 1911, p. 792–793]. For the sake of completeness, we reproduce the passages from Fleet's article. The first passage is from the *Saddasāratthajālinī*, a grammatical work by Nāgita:

cakkē pattē guṇaggaraṃ sake pana alappāyaṃ |
māghē māsē suniṭṭhitō taṃ sādhavō vicārentu ||

The verse informs that the work was completed in the month of Māgha, when Jinacakka, the era counted from the Buddha's death, had reached the year 2353 (*gu-ṇa-gga-ra*) and Sakkarāj, the common Burmese era, had reached 1130 (*a-la-ppā-yam*). But since the difference between the two eras is about 1183 years, the equation Jinacakka 2353 = Sakkarāj 1130, is wrong. Therefore, Fleet emended *alappāyam* as *aḷappāyam* and invested the consonant with the value 7, as in Malayālam script (see p. 43 above). Then Jinacakka 2353 = Sakkarāj 1170 give AD 1807 as the date of the work. But one

45 [Datta & Singh 1962, part I, p. 72]. [Sarma 2003, p. 48] repeats this without attribution to Datta & Singh.

can also emend the other expression *gu-ṇa-gga-ra* as *gu-ya-gga-ra* (2313) and then the equation would be correct. Therefore, this passage does not provide any firm evidence for the value of *ḷ* = 7, because *ḷ* does not occur in the original but only in Fleet's emendation.

The second passage is from the *Saṃvegavatthudīpanī* by Jāgara and reads thus:

sabbha-khattiya-dhammēna dhammarājēna yācito |
māpita-ratanapuṇṇēna katā saṃvegadīpanī ||
niṭṭhitō ēsō saṃpattē sakkarājē raṭṭhakkhayaṃ |
bhānuvakkhaṃ jinacakke pagguna-māsa-pañcamē ||

"This tells us that the work was finished on the fifth day of Phālguna, when Sakkarāj had reached the year *ra*(2)-*ṭṭha*(2)-*kkha*(2)-*ya*(1), i.e. 1222, and the Jinacakka had reached the year *bhā*(4)-*nu*(0)-*va*(4)-*kkha*(2), i.e. 2404. In Sakkarāj 1222 the given day, the fifth of Phālguna, i.e. Tabaung waxing 5, was 13 February, A.D. 1861." [Fleet 1911, p. 792–793].

It is obvious that neither passage contains the values $s = 5$, $h = 6$ and $ḷ = 7$, as stated by Datta and Singh. Consequently, this so-called "third variant" is no different from the standard system as (i). It is, however, worth while to investigate how this standard *Kaṭapayādi* notation came to be employed in Pali texts in Burma in the nineteenth century.

Likewise, the fourth variant postulated by Datta and Singh is based on some erroneous assumption. From the writings of K. V Sarma and from the number of examples he provided therein, and also from the examples cited in this paper, it is amply clear that the system as practised in Kerala reads numbers from the right to the left. Therefore one can speak of only two variants. The first is the real *Kaṭapayādi* system as described in the *Sadratnamālā* with the right-to-left arrangement. This is *the* Kerala system. The reverse arrangement adapted by Āryabhaṭa II was confined only to his work *Mahāsiddhānta*, just as the alphabetical notation devised by the first Āryabhaṭa was limited to his *Āryabhaṭīya*. Neither was emulated by others.[46]

5. CONCLUSION

The *Kaṭapayādi* system of numeral notation originated in Kerala sometime before the seventh century AD, and has been employed continuously

46 In his *Vedic Mathematics*, Jagadguru Swāmī Śrī Bhāratī Kṛṣṇa Tīrthajī, the pontiff of a Hindu religious order, employs an alphabetical notation which he styles "The Vedic Numerical Code". It is just the standard *Kaṭapayādi* system but read from the left to the right; cf. [Tīrthajī 1991, p. 194–195].

from the time of the *Candra-vākyas* until the present times,[47] not only in astronomical and mathematical literature, but in other genres as well. Closely associated with this notation are the mnemonic tables which have contributed to the development of the Kerala school of mathematics, and also the *Kali-ahargaṇa* method of dating events in terms of the civil days elapsed since the commencement of the Kali era. The alphabetical notation was prevalent, to a limited extent, in the other regions of peninsular India, notably in Tamilnadu.

From the number of anonymous definitions current in northern India, it is evident that the *Kaṭapayādi* system was well known there. Yet it is surprising that this very convenient tool was not employed in any scientific work. The only instances noticed so far are in Tantric or magico-religious texts and in two cases on astronomical instruments. Why the true potential of this notation was not made use of in any astronomical or mathematical text in northern India will remain one of the enigmas in the history of transmission of scientific ideas within India [Sarma 2009a, p. 221–223].

6. APPENDIX
DEFINITIONS OF THE *KAṬAPAYĀDI* SYSTEM

For the sake of convenience, I put together all the definitions of the *Kaṭapayādi* system which I have discussed above. It will be noticed that most of these are anonymous and must be part of floating oral literature.

1. Śaṅkaravarman, *Sadratnamālā* (1819 AD), 3.3:

nañāv acaś ca śūnyāni saṃkhyāḥ kaṭapayādayaḥ |
miśre tūpāntahal saṃkhyā na ca cintyo hal asvaraḥ ||

"*na*, *ña* and the vowels [standing alone denote] zeros; [the consonants] beginning with *ka*, *ṭa*, *pa* and *ya*, [denote severally] the numbers [1 to 9]; in a conjunct [consonant, only] the consonant in the penultimate [place, which is followed by a vowel, denotes a] number; and a consonant not attached to a vowel should not be considered [as a number]."

2. Anonymous:

kādi nava, ṭādi nava, pādi pañca, yādy aṣṭau.

47 [Sarma 2003, p. 55, n. 15] mentions three astronomical works which were composed in the second half of the twentieth century in Sanskrit with auto-commentaries in Malayalam, employing the *Kaṭapayādi* system of numeral notation, viz. P. A. Poduval, *Gaṇitaprakāśikā*, Kannur, 1952; Pulioor Purushottaman Namputiri, 2nd edn, Quilon, 1952; V. P. Kunjikanna Poduval, *Śuddha-Dr̥ggaṇitam*, Kozhikode, 1956.

"The nine [consonants] starting with *ka*, the nine starting with *ṭa*, the five starting with *pa* and the eight starting with *ya* [respectively denote the numbers 1 to 9].

3. Anonymous:

kādayo 'ṅkās (9) *ṭādayo 'ṅkāḥ* (9) *pādayaḥ pañca* (5) *kīrtitāḥ* |
yādayo' ṣṭau (8) *tathā prājñair gaṇakair buddhimattaraiḥ* ||

"The nine [consonants] starting from *ka*, the nine [consonants] starting from *ṭa*, the five [consonants] starting from *pa*, and the eight [consonants] starting from *ya*, are mentioned by highly intelligent astronomers [as the alphabetical symbols to represent the digits 1 to 9]."

4. Anonymous:

kaṭapayavargair nava-nava-pañcāṣṭa na-ña-jñāḥ śūnya-bodhakāḥ ||

"[The digits 1 to 9 are represented by the] nine [consonants] in sequence starting with *ka*, the nine [consonants] starting with *ṭa*, the five [consonants] starting with *pa*, and the eight [consonants] starting with *ya*; *na*, *ña* and *jña* indicate zero."

5. Anonymous:

kādir navāṅkāḥ nava ṭādir aṅkāḥ
pādiḥ śarā yādi bhavanti cāṣṭau |
nañau ca śūnye ca svarāś ca śūnyāḥ ||

"The nine [consonant] starting from *ka*, the nine [consonants] starting from *ṭa*, the five [consonants] starting from *pa*, and the eight [consonants] starting from *ya* [represent the digits 1 to 9]; *na* and *ña* [stand] for zero; and the vowels [represent] zero."

6. Anonymous:

kaṭapayavargabhavair iha piṇḍāntyair akṣarair aṅkāḥ |
nañe śūnyaṃ jñeyaṃ tathā svare kevale kathite ||

" Here the [nine] digits [are denoted] by the letters of the alphabet that belong respectively to the groups [of consonants] beginning with *ka*, *ṭa*, *pa*, and *ya* and which are the last members of the conjunct consonants (*piṇḍa*); *na* and *ña* should be known as zero; so also when a vowel is pronounced separately."

7. Āryabhaṭa II, *Mahāsiddhānta* (ca. 950 AD), 1.2:

rūpāt kaṭapayapūrvā varṇā varṇakramād bhavanty aṅkāḥ |
ñnau śūnyaṃ prathamārthe ā chede ai tṛtīyārthe ||

"The consonants (*varṇa*) starting from *ka*, *ṭa*, *pa*, *ya* represent the numerals from one (*rūpa*) [in succession] in the order of the consonants, *ña* and *na* [denoting] the zero. [The numerical expressions, when] separated [from each other, have] *ā* and *ai* [at their end] in the nominative [plural] and in the instrumental [plural respectively]."

8. Ṭoḍarmal, in his commentary (1762 AD) on the *Gommaṭasāra* of Nemicandra Siddhānta-cakravartin [Nemicandra 1974a, p. 223] [Datta 1930–31, p. 78, n. 4]:

kaṭapaya-purastha-varṇair nava-nava-pañcāṣṭa-kalpitaiḥ kramaśaḥ |
svara-ña-na-śūnyaṃ saṃkhyā mātroparimākṣaraṃ tyājyam ||

"Numbers (*saṃkhyā*) [are expressed] by the consonants, starting [severally] from *ka*, *ṭa*, *pa*, *ya*, which are respectively nine, nine, five and eight in number; the vowels, *ña* and *na* denote zero. The vowel symbols and the 'upper' (*uparima-akṣara*) consonants are to be ignored."

Here *uparima-akṣara*, the "upper letter", means the first consonant in a conjunct consonant; in Devanāgarī script the first consonant is written half, i.e. without the vertical stroke. In Kannada (and Telugu), the first consonant is written above and the second below it in the form of a special symbol. Though cited by Ṭoḍarmal of Rajasthan in the eighteenth century, the definition is clearly from the Jain community of Karnataka.

It may be noted that none of the definitions refer to the direction of enumeration.

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