# On the Rationale of the Maxim *Aṅkānāṃ Vāmato Gatiḥ*

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## ABSTRACT

Sanskrit texts on astronomy and mathematics arrange the digits in numerical quantities in a right-to-left sequence, starting from the unit's place, in accordance with the maxim *aṅkānāṃ vāmato gatiḥ*. However, nothing much is known about the origin or the rationale of this maxim. The noted historians of mathematics, Bibhutibhushan Datta and Avadhesh Narayan Singh, observe that "no explanation as to why the right to left arrangement was preferred in the word system is to be found in any of the ancient works." But in a Bengali article written earlier, Bibhutibhushan Datta himself had discussed the rationale of the maxim according to Gaṇeśa Daivajña and Nṛsiṃha Daivajña. This article, after a brief overview of the history of the *Bhūtasaṃkhyā* notation, presents three statements by Gaṇeśa Daivajña, Kṛṣṇa Daivajña and Nṛsiṃha Daivajña on the rationale of the maxim with English translations.

**Keywords:** *Bhūtasaṃkhyā, Kaṭapayādi,* Gaṇeśa Daivajña, Kṛṣṇa Daivajña, Nṛsiṃha Daivajña, word numerals.

## 1. INTRODUCTION

For expressing large numbers consisting of several digits, Sanskrit texts on astronomy and mathematics employ two kinds of notation. In the first notation, commonly known as *Bhūtasaṃkhyā* or word numerals, the digits 1 to 9 and zero are expressed by words, whereas in the second method called *Kaṭapayādi* they are represented by the consonants of the Sanskrit alphabet.

The first positive and datable occurrence of the *Kaṭapayādi* alphabetic notation is in Haridatta's *Grahacāranibandhana* which was composed in AD 683 in Kerala.

The notation must have been invented about this time or earlier in Kerala, where it was widely employed, not only in texts on mathematics and astronomy but in other kinds of works as well. It was also well-known in Tamilnadu, and to a limited extent in other regions of the peninsular India. It does not seem to have been employed in any mathematical or astronomical work in North India, but there are isolated cases of its use in magico-religious texts and on Sanskrit astronomical instruments.[1]

## 1.1 Bhūtasaṃkhyā Notation

The *Bhūtasaṃkhyā* notation, on the other hand, is older and was employed throughout India, and even outside. Here the digits 1 to 9 and zero, and also several double-digit numbers, are denoted by certain significant words which have numerical association. The symbolic words used in this system are derived from many areas, such as the human body (*netra, hasta* and *karṇa* for 2), mythology (*rāma* denotes 3 because there are three Rāmas in mythology, viz. Paraśu-rāma, Dāśarathi-rāma and Balarāma), ritual (*agni* denotes 3 because there are three fire altars in the Vedic ritual, viz. *Āhavanīya, Gārhapatya* and *Dakṣiṇa*), cosmology (*loka* = world= 3, for there are three worlds, viz. heaven, earth and the nether region; *gaja* = elephant = 8, for the flat earth is said to be supported by eight elephants, one at each side and each corner), poetic convention (*śara* = arrow denotes 5 because Kāma-deva, the god of love, is said to employ five flowers of the spring as his arrows) and different *śāstras* (*guṇa* stands for 3, because there are 3 *guṇas,* viz. *sattva, rajas and tamas*). *Chandaḥśāstra,* the science of prosody, contributed many names of the metres as symbolic words to denote certain numbers.[2]

However, the choice of the term *rūpa* for one or unity in the oldest of the sources to be discussed below, namely the *Vedāṅga-jyotiṣa* and *Chandaḥsūtra,* is somewhat intriguing, for none of the meanings of *rūpa* suggests anything unique. The Petersburg Sanskrit-German Dictionary proposes as one of the meanings "ein einzelnes Stück, Exemplar … daher Bez. der Zahl Eins," which the Monier-Williams Dictionary reproduces in English as: "A single specimen or exemplar (and therefore

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[1]For a comprehensive study of this notation, see Sarma (forthcoming).

[2]These terms are mainly for two-digit numbers: *atijagatī* metre has 13 syllables in each *pāda* and therefore represents 13, *śakvarī* (14), *atiśakvarī* (15), *aṣṭi* (16), *atyaṣṭi* (17), *dhṛti* (18), *atidhṛti* (19), *kṛti* (20), *prakṛti* (21), *ākṛti* (22), *vikṛti* (23), *gayatrī* (24), *atikṛti* (25), *utkṛti* (26), *jagatī* (48).

a term for the number 'one')". The Petersburg Dictionary cites in support of this meaning, Varāhamihira's *Bṛhatsaṃhitā,* 81.11: *sārdhās tisro guñjāḥ saptatimūlyaṃ dhṛtaṃ rūpam,* which would roughly mean that if a single pearl weighs (*dhṛta*) three and half *guñjās* (seeds of Abrus precatorius Linn), its price will be seventy (standard coins). But does *rūpa* here exactly signify "single specimen"? The commentator Utpala does not think so; he explains the passage thus: *yasya muktā- phalasya rūpaṃ pramāṇaṃ sārdhās tisro guñjā dhṛtaṃ tasya saptatimūlyam.*

I have consulted several colleagues on this term. Professor K. Ramasubramanian suggests the most plausible explanation: Every individual has his/her own distinctive, therefore, unique *rūpa,* "form, shape, appearance". It is thus the term *rūpa* came to stand for "one".

Modern writers have been using the term *Bhūtasaṃkhyā* for these symbolic words.[3] This designation was employed at the end of the fifteenth or the beginning of the sixteenth century by Sundararāja.[4] In his commentary *Laghuprakāśikā* on the *Vākyakaraṇa,* he states that in the *Vākyakaraṇa* the numbers are denoted by the *Kaṭapayādi* system and not by the *Bhūtasaṃkhyā* system:

*punar apīha pañcādhyāyyāṃ …sūryasiddhānta-bhāskarīyādi-śāstra-siddhabhū- tasaṃkhyā-parāṇi rūpanetrādīni ekadvitryādīny apahāya haridattādibhir aṅgīkṛtaparahitādi-śāstrokta-nyāyena kaṭapayādibhir eva saṃkhyābhidhīyate.*[5]

"Again here in this *Pañcādhyāyī* (i.e. *Vākyakaraṇa*), avoiding the *Bhūtasaṃkhyā* system as employed in the *Sūryasiddhānta,* Bhāskara's works and other texts (*śāstra*), where [words like] *rūpa, netra* and so on [stand for] one, two, three and so on, the numbers are denoted by the *Kaṭapayādi* system, which is adopted (*aṅgīkṛta*) by Haridatta and others and employed in texts (*śāstra*) like the *Parahita* (i.e. *Grahacāra-nibandhana*) and others."

No other Sanskrit text seems to have used this term *Bhūtasaṃkhyā* in this

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[3]For example, Pingree 1981:1 and Subbarayappa & Sarma 1985: 332-333 use *Bhūtasaṃkhyā.* It would be interesting to investigate who among the modern writers have made this designation popular.

[4]Pingree 1981: 48: "Sundararāja … was a native of Viprasadgrāma (Andaṇa-nal-lur) near Trichinopoly, in the late fifteenth or early sixteenth century."

[5]Sundararāja's *Laghuprakāśikā,* commentary on *Vākyakaraṇa* 2.7: *sarvatra saṃkhyā vihitā varṇaiḥ kaṭapayādibhiḥ,* pp. 36-37, esp. 37*.*

sense.[6] Māhavīra, at the beginning of his *Gaṇitasārasaṃgraha* devotes ten verses (1.53-62) to the word numerals under the title *saṃkhyā-saṃjñās* (symbols for numbers). Āryabhaṭa II employs word numerals in the fifteenth chapter of his *Mahāsiddhānta*, where he calls them *prasiddha-saṃjñās* (well-known symbols).

In modern times, Bibhutibhushan Datta and Avadhesh Narayan Singh have coined the term "word numeral" which is generally used now.[7]

### 1.1.1 Early Occurrences of the Word Numerals

Word numerals occur sporadically in the Vedic literature. The term *kṛta* is used to denote 4 in *Śatapatha-Brāhmaṇa* 13.3.2.1 and in *Taittirīya-Brāhmaṇa* 1.5.11.1. In *Kātyāyana-Śrautasūtra* 22.1.21-22 and *Lāṭyāyana-Śrautasūtra* 9.4.31, the names of the metres *Gāyatrī* and *Jagatī* represent 24 and 48 respectively.[8] The *Sulbasūtras* do not use these words but *Vedāṅga-jyotiṣa* does. This text employs generally common words for numbers, but in a few cases it uses word numerals as well; thus *rūpa* for 1 occurs in the Ṛk recension 31 = Yajus recension 23; *bhasamūha* (collection of the lunar mansions) for 27 occurs in Yajus recension 20.[9]

### 1.1.2 Word Numerals in Pingala's Chandaḥsūtra

The earliest text which employs these symbolic words systematically and quite extensively is Piṅgala's *Chandaḥsūtra,* which is generally placed in the second century BC. The roughly 328 *sūtras* in this text, divided into eight chapters, deal with the classification and definition of several Vedic and Classical Sanskrit metres

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[6]Hemādri, *Caturvargacintāmaṇi,* vol. 1, p. 135, quotes from the *Kāmikāgama* the line *mekhalaikāthavā tisro bhūtasaṃkhyāthavā priye* and explains *bhūtasaṃkhyāḥ* as *pañcasaṃkhyāḥ.* Thus here the expression merely means "the number five."

[7]Datta & Singh 1962: Part 1, 53-60. Plofker 2009: 47 states that "A different representation of decimal place value is revealed by a verbal notation called by medieval authors *bhūta-saṅkhyā* or 'object numbers,' here designated the 'concrete number system'." She does not explain the reason for this new designation.

[8]Ojha 1971: 121.

[9]Following Ojha 1971:121, Datta & Singh 1962: Part 1, 58 state that *aya* for 4 was employed in Ṛk recension 4 = Yajus 13, but this word is not met with in the latest edition of Kuppanna Sastry & K. V. Sarma. Datta & Singh write further that *guṇa* for 12 (!) was used in Ṛk 19 and *yuga* for 12 (!) in Yajus 25. In the latest edition, Ṛk 19 has *gaṇa* and not *guṇa;* Yajus 25 has *yuga* but it means there an era of 5 years and not the number 12.

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according to the number of syllables or of morae in them. Therefore, several *sūtras* contain words to indicate the number of syllables or of morae, the position of caesura (*yati*) and similar quantities. Here the words used to express the numbers are both common number words like *ṣaṭ* (6) or symbolic words like *ṛtu* or *rasa* (6), the two types occurring in equal proportion.

There are about a hundred occurrences of the common number words. These include cardinal numbers like *eka, dvi, …,* ordinals like *prathama, dvitīya,* and also words indicating groups like *trika* (tetrad) … *daśaka* (decade).

Likewise there are about a hundred occurrences of word numerals, such as *śūnya* (0), *rūpa* (1), *yuj* (2), *samudra, veda* (4), *indriya, kāmaśara, bhūta* (5), *ṛtu, rasa* (6), *ṛṣi, svara* (7), *vasu* (8), *dik* (10), *rudra* (11), *āditya, māsa* (12). These word numerals are mostly used singly to denote single-digit numbers. In some cases, two or more words are used together in a compound in the sense of $x$ or/and $y$. Thus *sūtra* no. 7.6 *svara-ṛṣayaḥ* means that in this metre there is one caesura after 7 (*svara*) syllables and another after another 7 (*ṛṣayaḥ*) syllables; 7.16 *ṛtu-samudra-ṛṣayaḥ* states there is one pause after 6 (*ṛtu*) syllables, another after 4 (*samudra*) syllables, a third after 7 (*ṛṣayaḥ*) syllables.

Three combinations of word numerals deserve attention. *Sūtra* no. 4.32 reads *vaitālīyaṃ dviḥ-svarā ayukpāde yug-vasavo 'nte r-l-gaḥ,* "*Vaitālīya* [is the meter in which] there are 14 (*dviḥ-svarāḥ*) [morae] in the odd feet (*ayukpāda*) and 16 (*yug-vasavaḥ*) [morae in the even feet], and at the end [of all four feet] there are one *ragaṇa* (*r*), one short syllable (*l*) and one long syllable (*g*)."

Here the two-digit number 14 is expressed by *dviḥ-svaraḥ* ($2 \times 7$) where *svara* is a symbolic word referring to the seven notes of music; and the number 16 by *yug-vasavaḥ* ($2 \times 8$), where *vasu* symbolically represents 8.

Again *sūtra* no. 4.42 reads: *gantā dvir-vasavo mātrāsamakaṃ l navamaḥ,* "*Mātrāsamaka* [is the metre where there are] 16 (*dvir-vasavaḥ*) [morae in each feet], and a long syllable (*g*) at the end [of each foot], and where the ninth [syllable] is short (*l*)." Here the two-digit number 16 is expressed by *dvir-vasavaḥ.*

In a notation with place value, 14 could have been expressed as *samudra-rūpa* or *veda-rūpa*, and 16 as *rasa-rūpa* or *ṛtu-rūpa.* The fact that Piṅgala did not employ such combination shows that he was not aware of place value.[10] Had he been writing in metrical verses, one could argue that he did not use such combination due to metrical constraints. But there are no metrical constraints in these prose

*sūtras* to prevent the use of expressions like *samudra-rūpa* or *veda-rūpa* for 14 and *rasa-rūpa* or *ṛtu-rūpa* for 16. Therefore, we are led to the firm conclusion that place value did not develop at the time of Piṅgala, although the use of symbolic words to represent numbers was developed to such an extent that Piṅgala employs this mode almost a hundred times.[11]

## 1.1.3 Word Numerals with Place Value

Word numerals with place value occur for the first time in Sphujidhvaja's *Yavana-jātaka*. At the end of this work, Sphujidhvaja states that he has completed the work in four thousand *Indravajrā* verses in *nārāyaṇaṅkendu-mitābda,* " the [*Śaka*] year measured by [the digits] *nārāyana* (1), [numeral] signs (*aṅka*, 9) and the moon (*indu*, 1)" (=AD 269/270).[12] In an earlier verse, he states that his work is a versification of Yavaneśvara's prose version, which the latter had composed in *viṣṇu-graha-abda,* "the year [measured by] Viṣṇu (1) and planets (*graha,* 7)," i.e. Śaka 71 = AD 149/150.[13]

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[10]I must admit that I have once argued, on the basis of Needham's statement that "zero symbol as part of the numerical system never existed and could not have come into existence without place value," that Piṅgala's use of *śūnya* presupposes place value in India; cf. Sarma 2003, Sarma 2009.

[11]Datta & Singh 1962: I, 58: "The use of word symbols without place value is found in the *Piṅgala Chandaḥ-sûtra* composed before 200 B.C. The principle of place value seems to have been applied to the word numerals between 200 B.C. and 300 A.D."

[12]*Yavanajātaka* 79.62:

> *sphujidhvajo nāma babhūva rājā*
> *ya indravajrābhir idaṃ cakāra* |
> *nārāyaṇāṅkendu-mitābda-diṣṭaṃ*
> *kṛtsnaṃ caturbhir mahimān sahasraiḥ* ||

[13]Ibid 79.60-61:

> *iti svabhāṣāracanābhiguptāṃ*
> *viṣṇu-grahābde*
> *mahūpamukhyair anudiṣṭatattvāṃ*
> *horārtha-ratnākara-vāk-samudrām* ||
> *sūryaprasād <ā>gata-tattvadṛṣṭir*
> *lokānubhāvāya vacobhir ādyaiḥ* |
> *idaṃ babhāṣe niravadyavākyo*
> *horārthaśāstraṃ yavaneśavaḥ prāk* ||

It may be noted that in 79.60*b,* more than half of the line is missing; therefore it is somewhat doubtful whether *viṣṇu-grahābde* was the original reading.

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Interestingly enough, in both the chronograms, the word *viṣṇu* and its synonym *nārāyaṇa* stand for unity. Such usage is not seen in later texts.[14] In another passage, Sphujidhvaja uses, apparently for the first time, zero (*bindu*) in the decimal place value system.[15] There are a few other word numerals in the text, but the great majority of numbers are expressed by common number words.

By the end of the fifth century the *Bhūtasaṃkhyā* notation with place value was fully developed and firmly established, so much so that Varāhamihira's *Pañcasiddhāntikā* expresses nearly all the numerical quantities in the entire work with this notation.[16] At the very outset, the work mentions the epoch of this work as *saptāśviveda,* i.e. Śaka 427 (= AD 505/6). One example should suffice to illustrate Varāhamihira's method of expressing numbers.

> *varṣāyute dhṛti-ghne nava-vasu-guṇa-rasa-rasāḥ syur adhimāsāḥ |*
> *sāvitre śara-nava-khendriyārṇavāśāḥ tithipralayāḥ ||*[17]

"According to the *Saura-siddhānta,* in ten-thousand (*ayuta*) [solar] years, multiplied by 18 (*dhṛti*) (i.e. 1,80,000 years), [there are] 66,389 (*nava-vasu-guṇa-rasa-rasa*) intercalary months and 10,45,095 (*śara-nava-kha-indrya-arṇava-āśā*) omitted lunar days." It may be noted that in all the numerical expressions, the digits are enumerated in the right-to-left order following the maxim *aṅkanaṃ vāmato gatiḥ.*

From this time onwards, all the subsequent works on astronomy and mathematics (in so far as they were not composed in Kerala or Tamilnadu) employed the *Bhūtasaṃkhyā* notation. Even outside the realm of Jyotiṣa proper, scholars began to express the numbers in this notation. In particular, the year of copying a manuscript or of the issue of an inscription is invariably expressed in the *Bhūtasaṃkhyā* notation. This aspect has been well documented.[18]

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[14]Sarma 2003 does not record *Nārāyaṇa*; he lists *viṣṇu* under 3.

[15]Pingree 1981: 1-2: "Sphujidhvaja also seems to be the first to use a symbol for *zero* (*bindu*) in the decimal place value system." The passage in question is 79.6 *ṣaṭ-pañcakāgre dviśate sahasraṃ teṣāṃ yuge binduyutāni ṣaṭ ca.* Here "*ṣaṭ-pañcakāgre dviśate*" means "two hundred, and 6 and 5 in front of it, i.e. 265, and *bindu-yutani ṣaṭ ca* means "six along with zero (*bindu*)", i.e. 60.

[16]The word numerals used in this text have been listed alphabetically by Pingree in his edition, Part I, p. 185 and by T. S. Kuppanna Sastry and K. V. Sarma on p. 372 of their edition.

[17]Varāhamihira, *Pañcasiddhāntikā,* 1.14.

[18]See especially, Sircar 1965: 228-233.

Along with Sanskrit language, Indian numerals and Indian calendar, the *Bhūtasaṃkhyā* notation was transmitted to Tibet and South-East Asia, where it was widely used to express the dates (or rather the years) in inscriptions.[19] Interestingly enough, the earliest inscription where the *Bhūtasaṃkhyā* notation was employed is to be found in Cambodia and not in India. In this inscription, the year of construction of a Śiva temple is mentioned as *rasa-dasra-bāṇa,* i.e. Śaka 526 (= AD 604/605) and the year of a subsequent consecration as *ṛtu-vāridhi-indriya,* i.e. Śaka 546 (=AD 624/625).[20]

### 1.1.4 Versified Lists of Word Numerals

As the word numerals began to be employed widely, attempts were made to prepare metrical lists or lexica of these words so that the beginners can learn them by heart. In the ninth century, at the beginning of his *Gaṇitasārasaṃgraha,* Māhavīra devotes ten verses (1.53-62) to enumerate the word numerals from 1 to 9 and 0.[21] Such verses also occur in the Telugu rendering of the same text which was done by Pāvulūri Mallana in the eleventh century. Strangely enough these verses are not in Telugu but in Sanskrit, and these are not the same verses as in Māhavīra's original.[22]

In the middle of the sixteenth century, Keśava Miśra composed a work on poetics entitled *Alaṅkāra-śekhara.* Here, in a small section entitled *saṃkhyāniyama-mayūkha,* Keśava provides a lexicon of word numerals for the poets to employ in their writings. There exist also several independent lexica of such terms. For example, the *Saṃjñā-nighaṇṭu* contains three sets of verses containing the *Bhūtasaṃkhyā*

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[19]On the spread of the word numerals outside India, see Coedès 1930-32, Gonda 1952, Jaquet 1835, Majumdar 1953, Majumdar 1985, Noorduyn 1993. I understand Anissa Oruzgan is making a special study of the word numerals in inscriptions in Cambodia and Java.

[20]Majumadar 1953:8-10. The relevant Sanskrit verse reads as follows:

> *dasa-dasra- śaraiś śakendravarṣe*
>
> *padam aiśaṃ vinibaddham iṣṭikābhiḥ |*
>
> *ṛtu-vārinidhāndriyaiś ca tīrthe*
>
> *(sa)lilasthāpanam akāri tena bhūpaḥ ||*

[21]These verses occur only in one manuscript, designated as M; they may or may not be part of the original as composed by Māhavīra.

[22]Pāvulūri Mallana. *Sārasaṃgrahagaṇitamu,* p. 13.

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words for 1 to 9 and zero.[23] Haridatta's *Gaṇitanāmamāla* is another unpublished lexicon of these and other mathematical terms.[24]

Al-Bīrūnī gives a list of words to represent numbers in his *India*.[25] Many modern works contain lists of these words.[26] One of the earliest scholars to study these terms was the Belgian indologist E. Jacquet[27] who discussed the word numerals in Sanskrit, Tibetan and Javanese and provided lists in these three languages in 1835. In Java the word numerals are known as *candra-sengkala* (from Sanskrit *candrasaṃkhyā?*). There are also versified vocabularies or lexica in Javanese language, containing word numerals derived from Sanskrit and also those coined independently in Java. These have been meticulously analysed by J. Noorduyn.[28]

## 2.0 The Maxim *aṅkānāṃ vāmato gatiḥ*

In both the *Bhūtasaṃkhyā*[29] and the *Kaṭapayādi*[30] notations, the words or the letters that denote numbers are arranged from the unit's place onwards in the right-to-left sequence, following the dictum *aṅkānāṃ vāmato gatiḥ,* "the movement of the digits [is from the right] to the left." But this order is the opposite of what is followed in writing. Therefore when decoding the numerical expressions, the sequence has to be reversed.

For example, at the beginning of his *Siddhānta-śiromaṇi,* Bhāskara states that the years elapsed between the commencement of the Kalpa and the beginning of the Śaka era are *go'drīndvadrikṛtāṅkadasranagagocandrāḥ,* i.e. cow (*go* 9), mountains (*adri* 7), the moon (*indu* 1), mountains (*adri* 7), *Kṛta* (4), numeral signs (*aṅka* 9), the Aśvins (*dasra* 2), mountains (*naga* 7), cow (*go* 9), the moon (*candra* 1).

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[23]A unique manuscript is with the Asiatic Society of Mumbai, see the Bibliography.

[24]Available in several manuscript copies, see Sen 1966:86-87.

[25]*Alberuni's India,* 1, pp.177-179. There are some errors in this list. E.g. *dadhi,* given for 4, should read *udadhi*; *khendu* is said to represent 10; actually it is not one word, but two, *kha* (0) + *indu* (1).

[26]For the most comprehensive lists, see Sarma 2003, especially Appendix I, pp. 59-69.

[27]See the Bibliography.

[28]See the Bibliography.

[29]Sircar 1965: 230 notes that in some inscriptions the right-to-left sequence is not observed. Instances of such irregular cases from Orissa are listed in Acharya 2002: 187-188.

[30]Āryabhaṭa II's *Māhāsiddhānta* of ca. 950 is an exception in that it uses the *Kaṭapayādi* from left to right.

These individual digits, when read in the reverse order, yield the number 1,972,947,179.[31] In the *Kaṭapayādi* notation this number can be expressed as *dhū-saṃ-yu-thā-va-dhū-ra-sā-dhā-kāḥ.*

But the scripts in which Sanskrit was written all proceed from the left to the right. Why should the words or letters standing for digits be arranged in the contrary direction? What is the origin and rationale of this practice? The practice must have originated about the same time as the place value, some time before the third century AD when the earliest recorded instances of place value occur in Sphujidhvaja's *Yavanajātaka.* As noted above, the *Yavanajātaka* contains two chronograms consisting of word numerals with place value, viz. *nārāyaṇa-aṅka-indu* for 191 and *viṣṇu-graha* for 71. In the first case, the first and last digits being the same, it cannot be said whether the sequence of the word numerals is from the right to left or the other way round. But in the second case, *aṅkānāṃ vāmato gatiḥ* is clearly followed.

As to the rationale of the maxim, Bibhutibhushan Datta and Avadhesh Narayan Singh observe in their celebrated *History of Hindu Mathematics* that "no explanation as to why the right to left arrangement was preferred in the word system is to be found in any of the ancient works."[32] They go on to say that "the following explanation suggests itself to us, and we believe it is not far from truth." The explanation they offer is that "the right to left arrangement is thus due to the desire of the mathematicians to look upon the process of formation of word chronogram as a sort of mathematical operation." What they mean by "mathematical operation" is the following.

In addition, subtraction and multiplication, we usually commence the operation at the unit's place and then proceed to higher powers in the right-to-left direction.[33] In fact, an anonymous text states *vṛddhi-hīne ca saṃyojye aṅkānāṃ vāmato gatiḥ,*

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[31]It may be noticed that, for the sake of metre, Bhāskara uses different synonyms for the same digit; thus *indu* and *candra* for the moon (1), *adri* and *naga* for mountain (7).

[32]Datta & Singh 1962: Part I, 62.

[33]According to Bhāskara, *Līlāvatī* 12, addition and subtraction can be done from the right to the left (*krameṇa*) or from the left to the right (*utkrameṇa vā*). Śrīdhara, *Pāṭīgaṇita* 18-19, states that in the *kapāṭa-sandhi* method, multiplication also can be done from the unit's place or from the highest place (*vilomagatyā 'nulomamārgeṇa vā*). However, the general practice in these three operations is to proceed from the unit's place.

"in multiplication, subtraction and addition, the digits proceed from the right to the left".[34]

Another explanation is that this right-to-left arrangement is an extension of the principle in the formation of numerical expressions of two digits, i.e. from eleven to ninety-nine, in Sanskrit and cognate languages where the number in the unit's place comes first and the number in the ten's place comes next, e.g. *ekā-daśa, dvā-daśa* in Sanskrit or *ein-und-zwanzig* und *neun-und-neunzig* in German.

However, there is a more fundamental reason. In fact, in a very valuable article, written in Bengali under the title "*Aṅkānāṃ vāmato gatiḥ: gaṇita vidhi,*" Bibhutibhusan Datta had lucidly discussed the rationale of the maxim, citing, among others, two passages from Gaṇeśa Daivajña and Nṛsiṃha Daivajña. Nṛsiṃha himself refers to Kṛṣṇa Daivajña on this issue.[35]

Briefly stated, the explanation offered by Gaṇeśa Daivajña, Kṛṣṇa Daivajña and Nṛsiṃha Daivajña, is the following. The identification of a notational place as the ten's place or the hundred's place is possible only with reference to the unit's place; that is to say, only when we proceed in the right-to-left direction and note that each notational place is ten's times higher than the previous notational place. We cannot start from the upper limit and proceed to the right, saying that each notational place is one-tenth of the previous place, for the upper limit is uncertain. Numbers being infinite, there cannot be any upper limit. On the other hand, the lower limit, i.e. the unit's place, is certain.

In other words, this arrangement is intrinsically connected with the principle of the decimal place value. A digit attains its real value from the place it occupies; a place receives its value in reference to the units' place. For example, if we encounter a number with many digits, such as the one mentioned above, viz. 1972947179, we have to count the notational places from the units' place in order to comprehend that the "1" at the left extremity has the value of one billion. That is why in modern notation, we insert commas (1,972,947,179) in order to be able to count the notational places quickly from the right.

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[34]*Saṃjñā-nighaṇṭu,* MS Asiatic Society of Mumbai, *f.* 4 verso.

[35]Datta does not seem to have had access to Kṛṣṇa Daivajña's commentary on Bhāskara's *Bījagaṇita,* which was published just about the time Datta was writing his article; cf. the Bibliography.

Therefore Gaṇeśa remarks: *ekādisaṃkhyānāṃ vāmakrāmam antareṇa gaṇanāyāḥ savyakramo na sambhavati,* "the left-to-right sequence (*savya-krama*) for counting (*gāṇanā*) (i.e. the verbal expression like 'three hundred and twenty-four') will not be possible without the right-to-left sequence (*vāma-krama*) [in assigning] the designations 'one' and so on."[36]

Since the explanations by these three savants have a fundamental bearing on the numerical notation and deal, so to say, with the philosophy of the decimal place value system, these passages will be reproduced below in full and translated into English, as far as possible, quite literally.

**2.1 Gaṇeśa Daivajña,** *Buddhivilāsinī* commentary (1545) on *Līlāvatī* 12.[37]

*ekādi-sthāna-sthitānām aṅkānāṃ krameṇotkrameṇa vā yogaḥ kāryo'ntaraṃ vā. katham. yathāsthānakam. sthānam anatikramya vartata iti yathāsthānam eva yathāsthānakam. ekasthānam ekasthāne yojyaṃ viyojyaṃ vā. evaṃ daśakasthāne daśakaṃ śatādisthāne śatādīty arthaḥ.*

*atvopapattiḥ. gaṇanākramaḥ savyakrameṇaiva bhāvyaḥ sarvatra, apasavyakramasya śiṣṭagarhitatvāt. ekādisaṃkhyānāṃ vāmakramam antareṇa gaṇanāyāḥ savyakramo na sambhavati.*

*yathā 1234. eṣām aṅkānām ekaṃ sahasraṃ dve śate daśakatrayaṃ catvāraś ceti savyakrameṇa gaṇanā syāt. lokair apy anenaiva krameṇocyate. na tu catvāras triṃśad dve śate sahasram ekam ity ucyate. api ca kāla-kīrtana-prayoge 'pi parārdha-kalpa-manvantara-yuga-vatsarādikaṃ deśakīrtane 'pi dvīpa-varṣa-khaṇḍādikaṃ ca sthūlāt sūkṣmam ity anenaiva krameṇocyate. evam ucyamāne gaṇanāyāḥ savyakrama-sthānānām aṅkakramo bhavati.*

*tasmād ekādi-sthānānāṃ vāmakrameṇaikādisaṃjñeti samācāraḥ.*

Translation:

"Addition (*yoga*) or subtraction (*antara*) of the digits (*aṅka*) occupying the notational places of units and so on (*ekādi-sthāna-sthita*) should be performed in the regular sequence (*krama*) or in the reverse sequence (*utkrama*). How? In accordance with the notational places (*sthāna*). [Grammatical analysis of the compound *yathāsthānam*]. [That is to say,] the digit occupying the unit's place

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[36]See below.

[37]Gaṇeśa Daivajña, *Buddhivilāsinī,* p. 13.

should be added to or subtracted from the digit occupying the unit's place. Likewise the digit occupying the ten's place [should be added to or subtracted from the digit] of the order ten (*daśaka*) and [that occupying] the hundred's place and so on [should be added to or subtracted from digits] of the order hundred and so on.

"Here is the demonstration (*upapatti*) of this [rule]. The sequence of counting (*gaṇanā-krama*[38]) should always be in the left-to-right sequence (*savya-krama*),[39] for the right-to-left sequence (*apasavya-krama*) is disapproved of by the learned (*śiṣṭagarhita*). [However,] the left-to-right sequence (*savya-krama*) in the verbal expression (*gaṇanā*) will not be possible without the right-to-left sequence (*vāma-krama*) [in assigning] the designations 'one' and so on (*ekādi-saṃjñā*).

"For example, [let us consider] 1234. The counting (i.e. verbal expression) (*gaṇanā*) of these digits will be made in the left-to-right sequence (*savya-krama*) as 'one thousand, two hundreds, three decades, and four.' People also speak in the same sequence. Nobody says 'four, thirty, two hundreds, one thousand.' Moreover, the [common] usage of mentioning time (*kāla-kīrtana-prayoga*) is in [the sequence of] *parārdha, kalpa, manvantara, yuga,* year and other sub-divisions; likewise a locality is mentioned [in the sequence of] *dvīpa, varṣa, khaṇḍa* and further sub-divisions; we speak in [the sequence proceeding] from the larger (*sthūla*) to the smaller (*sūkṣma*) [units]. Thus in spoken language (*ucyamāna*), the sequence of digits (*aṅka-krama*) for the verbal expression (*gaṇanā*) will be [that] of the notational places in the left-to-right sequence (*savya-krama*).

"Therefore, the notational places beginning with units receive the designations beginning with 'one' in right-to-left sequence (*vāma-krama*); this is the proper practice (*samācāra*) [on which the verbal expression (*gaṇanā*) is founded]."

Gaṇeśa's idea is as follows: *saṃjñā* (designation) of numbers begins with the smallest (unit's) place [because we cannot define "two" without "one", and ten's place without unit's place], while the *gaṇanā* (verbal expression of a number) begins, just by convention, with the largest notational place. Therefore *gaṇanā* is possible only after the *saṃjñās* have been assigned to the notational places.

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[38] By *gaṇanā,* Gaṇeśa means the verbal expression of a number with the positional value of each digit like "three hundred and twenty-four" and not mere counting like "one, two, three …"

[39] *Savya-krama* is literally the "sequence [from] the left"; but for greater clarity, this expression and *dakṣiṇa-krama* will be rendered here as the "left-to-right sequence"; likewise *apasavya-krama* and *vāma-krama* as the "right-to-left sequence".

**2. Kṛṣṇa Daivajña,** *Bījapallava/Navāṅkurā* commentary (ca. 1600) on the *Bījagaṇita*:[40]

*paramakāruṇiko bhagavān [svayambhūḥ] aticaturo navaivāṅkān sasarja yathā 1/2/3/4/5/6/7/8/9. atha cābhūṣṭasthānād vāmakrameṇa dvitīya-tṛtīyādi-sthānā- ny uttarottaram daśaguṇānāṃ saṃkhyānāṃ saṃjñābhir daśaśatādibhir asaṅketayat. prathamaṃ sthānaṃ caika-guṇa-saṃkhyā-sthānatvād ekasaṃjñayā.*

*tathā sati navaivāṅkas tatra sthāna-sambandhāt sthānāni vā tat-tad- aṅkasambandhād yathā svāntāntāṃ[41] saṃkhyāṃ jñāpayeyur iti sakalasaṃkhyā- vagamaḥ sugama iti. yathābhūṣṭasthāne niveśito 'yam aṅkaḥ 3 ekaguṇāyās tritvasaṃkhyāyā jñāpako bhavati. tato vāmato dvitīyasthāne niveśitaḥ svasaṃkhyā- yā daśaka-jñāpako bhavati. yathā daśakadvaya-jñāpako 'yaṃ[42] 20. evaṃ[42] vāmatas tṛtīya-caturthapañcamādi-sthāna-niveśito 'ṅka uttarottaraṃ daśaguṇānāṃ śata- sahasrāyutādīnāṃ yathāsvaṃ jñāpako bhavati.*

*tatrābhūṣṭa-saṃkhyāyā yathāsambhavam eka-daśaka-śatādy-abhāve tatsthāna- pūraṇārtham abhāva-dyotakāṅkaḥ śūnya-saṃjñako lipiviśeṣo niveśyate. yathā- ṣṭottaraśata-saṃkhyāyā daśakābhāvād dvitīyasthāne śūnya-niveśanam 108. yathā vāṣṭottarasahasrasaṃkhyāyāṃ daśaka-śatakayor abhāvād dvitīya-tṛtīyāsthānayos tat 1008.*

*anyathodāhṛta-saṃkhyayor[43] yathākramam aṣṭaka-śatakayor evā- ṣṭakasahasrayor[44] eva vā niveśe[45] 18 dvitīyasthāna-niveśitasya daśakajñāpakatvād aṣṭadaśatvaṃ pratīyeta nābhūṣṭa-saṃkhyā ata evātrāyuta-lakṣādīnām abhāve 'pi na[46] tat-sthāne śūnyaṃ niveśyate, tena vināpy-abhūṣṭasaṃkhyā-jñāpaka-sthānapū- raṇāt.*

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[40]There are two editions of this commentary. In the first edition in the Ānāndaśrama Sanskrit Series (ASS) the commentary bears the name *Navāṅkurā*, while it is styled *Bījapallava* in the second edition from Tanjore. There are slight differences in these two editions, which are noted below.

[41]Tanjore omits *svāntāntāṃ.*

[42]Tanjore *ekaṃ.*

[43]ASS *anayor udāhṛta-saṃkhyayor.*

[44]ASS omits *eva.*

[45]ASS *vā'niveśe,* meaning *vā aniveśe,* but it ought to be *niveśe* here. The *avagraha* is redundant.

[46]ASS omits *na.*

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*ato 'bhūṣṭa-saṃkhyāyām uttarāvadhi-bhūtāṅkasthānād dakṣiṇa-sthānānāṃ pūrakatvāt tatroktarītyā śūnya-niveśanam āvaśyakam. vāmasthānānāṃ tv apūrakatvād ānantyāc ca na tat tatheti.*

*nanv asti lipi-puṣṭa-savyakramaḥ*[47] *śiṣṭa-sammato māṅgalikatvād ādaraṇīyaś ca tat kathaṃ tam apahāyāpasavyakrama ādṛta iti cet. na. śata-sahasrāyuta-lakṣā-di-yuta*[48]*-saṃkhyāyā uttarottaram abhyarhitatvāt tatsavyakramasyocitatvād etatkramasya yuktatvāt.*

*na cābhyarhita-saṃkhyātaḥ savyakrāmārtham uttarāvadhitaḥ pradakṣiṇa-krameṇaiva dvitīyādi-sthānānāṃ saṃjñāstv iti vācyam. uttarāvadher abhāvāt. paricchinna-saṃkhyāsu tat-sattve 'pi tasyāniyatatvāt prathamāvadhes tu niyatatvāt tatsthānam ārabhya sthāna-saṃjñāyuktatarety alaṃ pallavitena.*

Translation:

"The great compassionate Lord [the Self-Born, i.e. Brahmā] ingeniously created just the nine digits, namely 1, 2, 3, 4, 5, 6, 7, 8, 9. Then he assigned to the second, third and other notational places, [which are counted] in the right-to-left sequence (*vāmakrama*) [starting] from any desired place, the designations (*saṃjñā*) ten, hundred, and so on, [designations which pertain to] numbers (*saṃkhyā*), successively ten-times higher [than the previous]. To the first notational place [he assigned] the designation 'one' because it is the place for the number whose value is obtained by multiplying itself with one (*eka-guṇa-saṃkhyā-sthāna*).

"For this reason, there are only nine digits, [so that they] by their combination with the notational places, or the notational places by their combination with the respective digits, would indicate (*jñāpayeyur*) the number (*saṃkhyā*) whose last (highest) [unit] is its last [notational place] (*svāntānta*), and thus the cognition (*avagama*) of all numbers becomes easy. For example, this digit 3 placed in any desired place (*abhūṣṭa-sthāna*) becomes the indicator (*jñāpaka*) of the number 3 which is multiplied by 1. The [digit] placed in the second notational place to the left (*vāmataḥ*) of that becomes the indicator (*jñāpaka*) of ten-fold (*daśaka*) of its own number, just as this 20 is indicator of two decades (*daśaka-dvaya*). Similarly the digit placed in the third, fourth, fifth and other notational places, as counted towards the left, becomes the indicator progressively of ten-fold, hundred-fold, thousand-fold, ten-thousand-fold of its own value.

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[47]ASS *lipiṣu savyakramaḥ*.

[48]ASS omits -*yuta*-.

"There, in a given number, when there is an absence (*abhāva*) of ones, tens, hundreds, and so on, as the case might be, in order to fill that place, a symbol (*lipiviśeṣa*) called 'empty' (*śūnya-saṃjñaka*), which denotes the absence [of a digit], is put [in the corresponding place]. For example, in the number one hundred and eight, owing to the absence of the decades, a zero is placed in the second notational place; thus 108. Or in the number one thousand and eight, owing to the absence of tens and hundreds, the same [zero is placed] in the second and third notational places; thus 1008.

"Otherwise, in the above-mentioned numbers, when only the [digits for] eight and hundred or the [digits for] eight and thousand are placed in the proper sequence, [both become] 18; because the digit placed in the second notational place indicates ten, this [number] would indicate the state of being eighteen and the desired number [108 or 1008] would not be understood. [Hence, zero has to be placed where necessary.] Therefore, even in the absence of ten-thousands (*ayuta*) and hundred-thousands (*lakṣa*), and so on here, a zero is not placed in those notational places, because even without it (i.e. the zero), the notational places required for expressing the desired number have been filled.[49]

"Therefore, in a given number, the notational places situated to the right of the notational place of the digit which constitutes the upper limit (*uttarāvadhi-bhūta-aṅkasthānāt*) require to be filled. Therefore, it is necessary to place zeros there in the manner indicated [above]. But the notational places to the left [of that upper limit] do not require to be filled; they are also endless; the above rule does not apply here (*na tat tathā*) (i.e. no zeros are placed to the left beyond the higher-most notational place which is given).

"Now the left-to-right sequence (*savya-krama*) is supported by the [manner of] writing (*lipi-puṣṭa*); it is approved by the learned (*śiṣṭa-saṃmata*) and is respected because it is auspicious. Why is it abandoned in favour of the right-to-left sequence (*apasavya-krama*)? If [you were to argue] thus, the answer is no. [You may argue further that] in a number containing hundred, thousand, ten-thousand, hundred-thousand, and others, the next one being always higher and respectable (*abhyarhita*), the left-to-right sequence (*savya-krama*) is proper (*ucita*) and this sequence is employed [in actual speech].

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[49]That is to say, in the number 1008, no zeros need to be put in the notational places to the left, even though the notational places of ten-thousands (*lakṣa*), hundred-thousands (*koṭi*) and so on are empty, because such zeros would be endless, and because the number 1008 will be understood even without the zeros to the left.

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"[You] cannot say that, for the sake of left-to-right sequence starting from a high and respectable number (place), the designation of the second and higher notational places be made by left-to-right sequence (*pradakṣaṇa-krama*) from the upper limit (*uttarāvadhi*); because there is no upper limit [of notational places]. Although the [upper limit] exists in particular (*paricchinna*) numbers, it being uncertain (*aniyata*), the first limit (*prathamāvadhi*) being certain (*niyata*), it is more appropriate to assign the designations to the notational places starting from that notational place (i.e. the unit's place). There is no need of further elaboration (*alaṃ pallavitena*)."

**3. Nṛsiṃha Daivajña,** *Vārttika* (1621) on *Siddhānta-śiromaṇi,* Madhyāma-dhikāra, Kālamānādhyāya, 28-29,[50] (explains why Bhāskara mentions the digits in the number of years elapsed between the commencement of the Kalpa and the beginning of the Śaka era in right-to-left sequence):

*atrāṅkānāṃ vāmato gatiḥ ekasthānād daśaka-sthānādi-vinyāsasya vāmakrameṇaivādyāiḥ saṅketitatvāt. eka eva dvy-ādi-navāvasānāvṛttibhir dvitvā-disaṃkhyāṃ labhate. evaṃ navaivāṅkāḥ saṅketitāḥ.*

*punar eka eva[51] daśaguṇottaro daśaka-śata-sahasrādi-saṃkhyāṃ labhate. yatra navāvasānāvṛtta eko vinyasyate tad eka-sthānam. yatra navāntāṅkāvṛtta eva daśa-guṇottaro vinyasyate tad daśakādi-sthānam ity āhuḥ.*

*sthānānantyāt saṃkhyāyā ānantyam. sthānānāṃ yo 'vaśyaṃ mantavya uttarā-vadhis tasya parārdha iti saṃjñā kṛtā. ācāryeṇa parārdhasyāṣṭādaśa sthānāny uktāni. kvacic chāstrāntare 'dhikāny uktāni.*

*śūnyaṃ nāmābhāvas tad api saṃkhyāntargatam eva saṃkhyā-dyotakatvāt. eka eva sahasra-saṃkhyāṃ kathaṃ dyotayed yady eka-daśa-śata-sthāneṣv aṅkānām abhā-vāc chūnya-niveśo na syāt.*

*nanv ekādi-sthāna-sthitānām aṅkānām abhāve yadvac chūnya-niveśena lakṣā-di-saṃkhyāvabodhas tadvat parārdhādi-sthānānām abhāve śūnya-niveśenāpi syāt. yuktaś ca vāma-kramād dakṣiṇa iti cet kim atra vaktavyam.*

*uktam evātra bīja-gaṇitaṃ vyākhyātavadbhiḥ kṛṣṇa-daivajñair uttarāvadher abhāvāt paricchinna-saṃkhyāsu tat-sattve 'pi tasyāniyatatvāt prathamāvadhes tu niyatatvād iti. uttarāvadheḥ pradakṣiṇa-krameṇaiva dvitiyādi-sthānānāṃ saṃjñā-stv iti tatrāpīdam evottaram.*

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[50]Nṛsiṃha Daivajña, *Vārttika,* p. 27.

[51]The printed text erroneously reads *iva* here.

*abhyarhita-sthāna-sthasya paṅktau pūrva-niveśas tad-adhaḥ-sthitasthānasthā-nāṃ savya-krameṇa sthāpanam ucitaṃ lokeṣu tathā dṛśyate tat tv ekasthānād vā-makrameṇa daśakādi-sthāna-vinyāsenopapadyate.*

*athavā paramāṇvādy adhikṛtya dvy-aṇukādi-saṃjñāḥ kriyante tadvad ekasthā-nam adhikṛtya daśakādi-sthāna-saṃjñā-karaṇe na kaścid doṣaḥ. ekādi-sthānasā-dhyatvād daśaka-sthānādīnām uttarottara-saṃkhyāyāṃ pūrva-pūrva-saṃkhyāyāḥ sāttvat[52].*

*tasmād ekam abjaṃ navārbudāni sapta koṭyaḥ prayuta-dvayaṃ lakṣanavakaṃ catvāry ayutāni sahasra-saptakaṃ śatam ekaṃ sapta daśakā[53] nava ceti gatāḥ śakā-dau.*

Translation:

"Here the digits proceed [from the right] to the left, because the ancients have laid down that the notational places are to be arranged from the unit's place (*ekasthāna*) to the ten's place and so on, only in the right-to-left sequence (*vāmakrameṇa*). Unity (*eka*) itself, when repeated (*āvṛtti*) twice to nine times, attains the numerical value (*saṃkhyā*) of two and so on. Thus only nine digits have been laid down [by the ancients].

"Again, unity itself, when progressively increased each time ten-fold (*daśaguṇottara*), obtains the numerical value of ten, hundred and so on. Where unity, after being repeated [once] to nine times, is placed that is the notational place of units (*ekasthana*). Where [unity], after being repeated [once] to nine times and then progressively increased ten-fold (*daśa-guṇottara*), is placed that is the notational place of tens, and so on (*daśakādi-sthāna*); thus state [the ancients].

"Because the notational places are unlimited, the numbers are unlimited. The upper limit (*uttara-avadhi*) of the notational places, which must necessarily be taken into account, is given the designation *parārdha.* The [venerable] teacher (i.e. Bhāskarācārya) mentioned that *parārdha* has eighteen notational places. In some other texts more [notational places] are mentioned.

"Zero (*śūnya*) means 'absence' (*abhāva*); this is also included among the numbers because it [too] indicates numbers. How can 'unity' indicate the number thousand, when the zero is not placed in the notational places of ones, tens and hundreds owing to the absence of digits [in those notational places]?

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[52]The printed text erroneously reads *satvāt* here.
[53]The printed text erroneously reads *daśakāḥ* here.

"Now, if you say that, just as we get the cognition of the number hundred-thousand (*lakṣa*) etc. by putting zeros where there is no digit in the notational places beginning with unity, even so there would be [the cognition of the same number] by putting zeros where there is no [digit in the] notational places beginning with *parardha,* and that the left-to-right sequence (*dakṣiṇa*) is more appropriate than the right-to-left sequence (*vāmakrama*), what can be said here [against such view]?[54]

"In his commentary on the *Bījagaṇita,* Kṛṣṇa Daivajña has indeed remarked that, when there is no upper limit, even though it exists in particular numbers (*paricchinna-saṃkhyā*), the upper limit being always uncertain, and because the first limit (*prathama-avadhi*) being certain, [it is proper to arrange the digits from the lower limit in the right-to-left sequence]. If you say, let the second and other notational places be designated by starting from the upper limit and proceeding only in the left-to-right sequence (*pradakṣiṇa-krama*), even then the answer is the same.

"First placing in the line [the digit] in the highest notational place (*abhyarhitasthānastha*), then placing (*sthāpana*) [the digits] which are situated in the notational places situated below that in the left-to-right sequence (*savya-krama*) is appropriate, because the same is seen in the common practice (*lokeṣu*); but that becomes possible only by starting from the unit's place and assigning the notational places of tens and others in the right-to-left sequence (*vāma-krama*).

"Or, just as, taking the *paramāṇu* etc. as the basis, terms like *dvi-aṇuka* and such terms are coined, even so when we take the notational place of units as the basis, and coin designations of the second and other notational places, there is no harm, because the notational places beginning with tens have the state of being established by the notational places beginning with units, because the preceding number resides in the succeeding number.[55]

"Therefore one billion (*abja*), nine hundred-millions (*arbuda*), seven ten-millions (*koṭi*), two millions (*prayuta*), nine hundred-thousands (*lakṣa*), four ten-thousands

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[54]The opponent's idea is as follows. The number *lakṣa* may be expressed as 000000000000100000 (with 12 zeros before '1' and 5 zeros after that) in the 18-decimal-place system. Therefore, if the highest place is fixed at the 18th notational place named *parārdha*, *lakṣa* can be expressed as 0000000000001 as well as 100000.

[55]That is to say, the number "two" resides in, or part of, "three"; "ten' resides in, or part of, "hundred"; etc.

(*ayuta*), seven thousands, one hundred, seven tens (*sapta daśaka*) and nine [years] have elapsed [from the commencement of the Kalpa] up to the beginning of the Śaka era (1,972,947,179)."

## ACKNOWLEDGEMENTS

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