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THE PRATYAYAS: INDIAN CONTRIBUTION TO COMBINATORICS*

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The paper is an English translation of Ludwig Alsdorf's original version in German. It presents texts and translations, with mathematical relationale of Hemacandra's *Chandonuśāsana*. Piṅigalas *Chandaḥśāstra*, *Pratyaya* chapter of Bharata's *Nāṭyaśāstra*, and a few other sources (*Prākṛtapaiṅgala*, Dāmodara's *Varṇabhūṣaṇa*. Kedāra's *Vṛttaratnākara* etc.). The treatment is critical and analytical as far as it concerns the previous studies by Weber, Cantor and other scholars. The history of the development of the subject and its assessment with Pascal's work have been presented. The paper carries a good bibliography, with notes and references.

I. INTRODUCTION

The classical prosody of India employs the names of Vedic metres as the collective designations for all the metres having a specific number of syllables. Thus, e.g. each metre of 8 syllables is called *anuṣṭubh*, of 11 syllables *triṣṭubh*, of 12 syllables *jagatī*, and so on. Within each of these groups, there are a certain number of individual metres which follow a fixed pattern. Thus the metres called *Indra-vajrā, Upendravajrā, Rathoddhatā, Vātormī, Śālinī* and several others belong to the *triṣṭubh* group. The patterns of these metres, however, do not exhaust all the possibilities of a verse quantified in 11 syllables.

Owing to the peculiar way of Indian thinking, the following question arises automatically, and it did arise at quite an early stage: how many and what type of patterns would theoretically be possible within such a group?

There is another factor that favours such a query. It is well known that all the syllabic metres are divided, for the sake of scansion, into groups of three syllables each (*gaṇa*) which are designated with the letters *m, n, r, s* etc. There are 8 such groups — neither less nor more — because out of 2 quantities, viz. long syllables and short syllables,[1] only 8 different groups of three can be formed, or, to use the terminology of modern mathematics, there will be 8 variations of 2 elements taken 3 at a time, allowing repetition. In the case of these trisyllabic groups, all the possible patterns had to be taken into consideration. Consequently there arose the desire to arrange these patterns

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according to some method that would ensure the inclusion of all the existing possibilities and also the non-existence of further possibilities.

Again from here one can easily reach the stage to look for the number of possible patterns for a larger, and eventually any arbitrary, number of syllables, and then to seek a method of systematic arrangement of these possible patterns. When one has such a systematic table in front of oneself — it assumes gigantic size even for a moderate number of syllables — one is then led easily to a series of further problems; for instance, how to determine the serial number of a real metre within the system, without first having to construct the entire table and then count the serial number? Thus one thing leads to another. Mathematical speculation, once awakened, goes on probing into things that have absolutely nothing to do with the prosody, and thus the theory of the so-called *Pratyayas* was born. Its foundations can already be seen in the *Piṅgala-Chandaḥśāstra;* it was developed further and elaborated in later times, and became a firm constituent of most of the text books on Indian prosody.

Here we see a process which is, and ought to be, typical for the history of mathematical thinking in general. After all, it must always be the case that, to put it paradoxically, pure mathematics develops from applied mathematics. Euclidean mathematics grew out of the art of field-measurement; the complicated modes of constructing sacrificial altars in India gave the first impulse for dealing with geometrical problems. Mathematical speculation receives an impetus at some point or other, and then it goes on searching and probing out of pure mathematical interest, far beyond the problems posed at the starting point. In a like manner, the theory of *Pratyayas* developed into a formal system of combinatorics about which a later theoretician like Hemacandra (cf. infra) observes that, to a large extent, it has no practical utility for prosody. It is true that the decisive step towards pure mathematics was not taken here; even the later additions just aim at extending the *Pratyaya* theory to all the varieties and sub-varieties of the metrical system. Thus, under syllabic metres, special rules were formulated for *arthasama* and *viṣama vṛttas;* writers on Prakrit prosody felt the need for discussing the moric metres and so on; these are things about which the *Piṅgala-Chandaḥśāstra* had no notion yet.

But, on the whole, we would do injustice to he *Pratyaya* theory if we were to treat it as an abstruse and useless appendix to prosody. In fact, this theory forms an interesting chapter of Indian mathematics, a collection of at times extraordinarily ingenious and clever solutions to problems. It will be worthwhile to study these problems from the mathematical point of view. That the astonishing feats of Indian theoreticians are something more than useless jugglery of numbers can best be seen when we note that often the same problems are solved in modern mathematics also. For example, when a text-book on elementary analysis asks us to determine the 2311st lexicographic complex of the letters *e, h, i, l, l, m, w* (solution: "Wilhelm") or conversely when it teaches a method for finding out which lexicographic complex of the letters *e, h, i, l, l, m, w,* constitutes "Wilhelm" (solution: serial number 2311),[2] it is just the same as what Indians teach under the *Pratyayas* called *Naṣṭa* and *Uddiṣṭa.*

Not everything that is taught under the *Pratyayas* is equally interesting. From the mathematical point of view, many of the individual problems are fruitless deviations. It could not have been otherwise at this, after all, primitive stage when mathematics had not been elevated to being an end in itself but was still associated with an alien theme. On the other hand, the investigation of another *Pratyaya* which Hemacandra styles *Sarvaikādi-ga-la-kriyā* yeilds the astonishing fact, to which no attention has been paid so far, that Pascal's famous triangle was known to India centuries before Pascal's time. It is probable that the writers on prosody who speculated on these matters had often no real knowledge of the actual mathematical significance of the numbers with which they operated; it is also possible that, at least at the beginning, most of the methods were derived in a purely empirical manner (in the case of several methods, we can trace the gradual improvements and see how theoretical perception became sharper at each step). All this does not detract from the mathematical significance of the *Pratyayas*, and one cannot but admire most of the *Pratyaya* methods as specimens of the great mathematical, and especially algebraic, skill of Indians. In fact, on the basis of the little he could glean from Weber's work — about which we shall speak presently —, M. Cantor says in his *Vorlesungen über die Geschichte der Mathematik* (vol. I, 2nd end., p. 619 f.): "all these are things that were certainly not known to any Greek in such perfection."

The only person to study the *Pratyayas* so far has been Alfred Weber. While translating the *Piṅgala-Chandaḥśāstra*, he had to deal with the chapter on the *Pratyayas* as well. This he did most thoroughly on pp. 425-457 of the eighth volume of the *Indische Studien*,[3] as also previously, by utilizing the later theories of Kedāra and the rules in the *Agnipurāṇa*. However, his treatment needs corrections and additions. He misunderstood some parts totally; in several other cases he admitted that much remained obscure to him. Again, he discussed only in parts, that too briefly, the mathematical basis of the methods described. Finally, his study did not touch upon the extensions and additions of the later period, especially the extension of the theory to moric metres.

I myself was led to the study of the *Pratyayas* while working on Hemacandra's *Chando'nuśāsana*. This text which is extemely important for Prakrit and more particularly Apabhraṃśa prosody and which became known just a few years ago through an Indian edition[4] in its complete form, i.e. along with Hemacandra's own commentary, devotes the entire last chapter to the *Pratyayas*. Since Hemacandra's treatment distinguishes itself through clarity, systematic construction and thoroughness and is thus best suited as an introduction to the subject, I shall begin with a full translation of the seventh chapter of the *Chando'nuśāsana*. Next, I shall discuss the material presented by Weber item by item, and take this opportunity to correct his errors and also to discuss the mathematical basis of the individual *Pratyayas*. The results obtained up to this stage will enable us to attempt a reconstruction — a least partially- of the *Pratyaya*-chapter of Bharata's *Nāṭyaśāstra*, a chapter that is hopelessly mutilated and reproduced in an impossible manner even in Regnaud's monograph.[5] This will be followed, finally, be a discussion of the *Pratyayas* for moric metres. Hemacandra's treatment, which occupies a special position like his Prakrit and Apabhraṃśa prosody, will be supplemented by other sources: *Prākṛtapaiṅgala*, Dāmodara's *Varṇabhūṣaṇa* and Bhaṭṭa Nārāyaṇa's commentary on the *Vṛttaratnākara* of Kedāra. This commentary, written in *Saṃvat* 1602 (= A.D. 1556/7) is the latest of our sources, and it discusses in greatest detail everything that has ever been said on this subject.[6]

II. Hemacandra's Chando'nuśāsana

Now I shall reproduce the seventh chapter of Hemacandra's *Chando'nuśāsana* in the following manner: I translate the *sūtras* and complete the sentences on the basis of the commentary. This will be followed — instead of a literal translation of the commentary — by a free and, as far as possible, brief and lucid exposition which presents Hemacandra's theories faithfully and completely. I think this procedure will be more to our purpose. Additions, explanations and comments by me will be enclosed in square brackets.

1. *atha prastārādayaḥ ṣaṭ pratyayāḥ.*

"Now follow the 6 *Pratyayas,* namely *prastāra etc.*" The commentary explains *prastāra* as *vṛttānāṃ vistarato vinyāsaḥ* and cites the following list:

> *Prastāro* (1), *naṣṭam* (2), *uddiṣṭaṃ* (3), *sarvaikādigalakriyā* (4) ¦
> *saṃkhyā* (5), *caivādhvayogaś* (6) *ca, ṣaḍ ete pratyayāḥ smṛtāḥ* ||

2. *prāk-kalpādya-go 'dho laḥ; param: upari samaṃ, prāk pūrvavidhir; iti samaya-bheda-kṛd-varjaṃ prastāraḥ.*

"(One should always change) the first long syllable of the preceding form into a short syllable in the succeeding (form). Of the remaining (syllables), those following (the changed long syllable remain) unchanged and those preceding (it assume) their original shape again. Thus, by avoiding (the forms) which go against the rules (of the metre concerned), the *prastāra* is constructed."

In the case of the *sama-vṛttas,* we begin the process by taking G for all the syllables of the *pāda,* e.g. ——— for a tri-syllabic *pāda.* Here the first G is changed into L, the following syllables remain as they are. Thus we get ∪——. Again, in this form the first G is changed, the next syllable remains as it is and the preceding one, however, assumes its original shape, i.e. the shape it had in the starting form (*prathame vikalpe*), and so on. [From the practical point of view, all the syllables preceding the changed one are written as G.]

—	—	—
U	—	—
—	U	—
U	U	—
—	—	U
U	—	U
—	U	U
U	U	U

Fig. 1

In the case of the *artha-sama-vṛttas,* the process must be extended to this half-verse. If we assume that the half-verse consists of 2 *pādas* of 2 syllables each, the *prastāra* then will be as shown in Fig. 2. However, of these forms, nos. 1,6,11 and 16 violate the definition of the *artha-sama-vṛtta,* namely that the 2 *pādas* of each half-verse must be unequal. Therefore, these forms are excluded as *samaya-bheda-kṛt.*

1	—	—	—	—
	U	—	—	—
	—	U	—	—
	U	U	—	—
	—	—	U	—
6	U	—	U	—
	—	U	U	—
	U	U	U	—
	—	—	—	U
	U	—	—	U
11	—	U	—	U
	U	U	—	U
	—	—	U	U
	U	—	U	U
	—	U	U	U
16	U	U	U	U

Fig. 2

1	—	—	—	—	—	—	—	—
18	U	—	—	—	U	—	—	—
35	—	U	—	—	—	U	—	—
52	U	U	—	—	U	U	—	—
69	—	—	U	—	—	—	U	—
86	U	—	U	—	U	—	U	—
103	—	U	U	—	—	U	U	—
120	U	U	U	—	U	U	U	—
137	—	—	—	U	—	—	—	U
154	U	—	—	U	U	—	—	U
171	—	U	—	U	—	U	—	U
188	U	U	—	U	U	U	—	U
205	—	—	U	U	—	—	U	U
222	U	—	U	U	U	—	U	U
239	—	U	U	U	—	U	U	U
256	U	U	U	U	U	U	U	U

Fig. 3

For the *viṣama-vṛttas,* the whole stanza must be taken into account. For instance, if it consists of 4 *pādas* with 2 syllables each, we should begin with ——/——//——/——. When the whole process is completed, there will be 256 forms in the *prastāra*. Of these, all such forms that exhibit the same half-verses or quarter-verses should be excluded as non-viṣama. These are, as the commentary explains, the first and every seventeenth, counting from the second up to the last, i.e. nos. 1, 18, 35, 52, 69 etc. up to 256. These are altogether 16 forms, of which four, namely nos. 1,6,11,16, are *sama-vṛttas* and the rest *artha-sama-vṛttas* (cf. Fig. 3 taken from the commentary[7]).

The process described above will now be extended, in a somewhat forced way, to the *mātrā-vṛttas* as well. We take the *Āryā* as an example. Here also, one starts with all Gs. Then the first G of the first *gaṇa* is shortened and the following Gs remain as they are. According to the rule *prāk pūrvavidhiḥ*, the number of the morae in the first *gaṇa* has to be maintained. Therefore, one mora has to be added at the beginning of the line (*ekā mātrā prāk nyasanīyā*). Then the second G of the first *gaṇa* is shortened (see Fig. 4), and the L achieved through shortening in the previous step must revert to a G. Thus the first *gaṇa* becomes — u. This cannot be filled up as u — u, it would violate the rule according to which an odd *gaṇa* should not contain an amphibrach (*samaya-bheda*). Hence, in step 3, the first *gaṇa* should assume the form — u u. Now the first G is changed here and all the following remain as they are. Thus we have u u u / — — / ... . The only possible way of completing this is u u u u / — — / ... .

1	— —	— —	
2	(u) u —	— —	
3	— (u) u	— —	
4	(u) u u u	— —	
5	— —	(u) u —	— — . . . .
6	u u —	u u —	— — . . . .
7	— u u	u u —	— — . . . .
8	u u u u	ů u —	— — . . . .
9	— —	u — u	— — . . . .

Fig. 4.

Gaṇa	Forms
1	4
2	5 × 4 = 20
3	4 × 20 = 80
4	5 × 80 = 400
5	4 × 400 = 1600
6	2 × 1600 = 3200
7	4 × 3200 = 12800
8	1 × 12800 = 12800

Fig. 5.

Next, the first G of the second *gaṇa* has to be changed. Following the process employed in step 1, we get u u — for the second *gaṇa* but the first *gaṇa* must be changed to — — at the same time. Since again the very first available G, i.e. the first G of the first *gaṇa*, has to be changed, the first *gaṇa* assumes the forms 2-4 in steps 6-8 once again, the only difference being that the second *gaṇa* has the form u u — instead of — —.

In step 9, the second G of the second *gaṇa* has to be changed. Since the second *gaṇa* can be an amphibrach, u u — becomes now u — u, unlike in step 3. The first *gaṇa* is changed at the same time into — — and goes through its 4 forms once again, with the amphibrach as the second *gaṇa*, and so on and so forth.

Since the second and fourth *gaṇas* can have 5 forms each, the third, fifth and seventh 4 each, the sixth 2 (u u̲ u̲ u), and since the anceps-syllable of the eighth *gaṇa* is always treated as a G, that is to say, has only 1 form, the number of possible forms for the first half of the *Āryā* will be as shown in *Fig. 5*.

[If the process is continued for the second line of the *Āryā*, which the commentary did not do, there will be 81,920,000 forms for the whole stanza.]

A similar process has to be employed for all the other *mātrāvṛttas*.

The next *sūtra* teaches a second variety of *prastāra* for the *sama-vṛttas*.

3. *g-lāv adho 'dho, dvir dvir ataḥ.*

"One should write) one G and L each (always alternately) one below the other; thereafter (in each succeeding column) always double (the number)."

That is to say, in the first column (*paṅki*) there will always be one G and one L alternately, in the second always $\begin{smallmatrix}-\\-\end{smallmatrix}$ and $\begin{smallmatrix}\cup\\\cup\end{smallmatrix}$ alternately,

in the third column $\begin{smallmatrix}-\\-\\-\\-\end{smallmatrix}$ and $\begin{smallmatrix}\cup\\\cup\\\cup\\\cup\end{smallmatrix}$

alternately, and so on, until the number of the possible variations is reached (*yāvat saṃkhyāparimāṇam*). With 3 syllables, 8 variations were possible, and we obtain the same table as in the case of the first *prastāra*, viz. Fig. 1.

[In view of the fact that these *prastāra* tables for larger number of syllables or for metres like the *Āryā* run into innumerable variations, there arise now two problems. How can we, without first having to write down the entire *prastāra*, state 1) which is the form of any given number in the *prastāra* series, and 2) conversely, which serial number has a given form? The unknown form of a given serial number is called *naṣṭavṛtta*, a given form whose serial number is sought is called *uddiṣṭavṛtta*.]

4. *naṣṭāṅkasya dale laḥ, saikasya gaḥ.*

"(Continuous) halving of the number of the form sought (when the division leaves no remainder) result in a L; (when the division leaves a remainder, halving) the number increased by 1 (yields) a G."

Divide the given serial number by 2. If there is no remainder, write down one L; but if the number is odd, add 1 to it before division and write down one G. Divide the result once again by 2, and continue the process so long until the number of the syllables sought is reached.

Example: Wanted the 5th form of the tri-syllabic *pāda*. $5 \div 2$ leaves a remainder, so we write a G. $(5 + 1) \div 2 = 3$. $3 \div 2$ again leaves a remainder, hence we write down a G. $(3 + 1) \div 2 = 2$. $2 \div 2$ leaves no remainder, hence a L is written down. Thus the form sought is — — ∪. [The division can be continued as long as one likes by adding 1 to the last 1 and by dividing the resulting 2 by 2. Thus, as soon as we reach a 1, further continuation of the process will yield just Gs.]

In the case of the *mātrā-vṛttas*, the process is as follows:

5. *naṣṭāṅke gaṇair hṛte śeṣa-saṃkhyo gaṇo deyo, rāśi-śeṣe labdhaṃ saikam.*

"Divide the number of the required form (successively) by the (numbers of the possible forms of individual) *gaṇas*, and (after each division) write down (that form of) the *gaṇa* whose serial number is the same as the remainder; [when there is no remainder, the divisor itself is treated as the remainder]; if there is a remainder (after the division), the quotient should be increased by 1 (before the next division)."

The commentary cites the following example, which I give below in a tabular form, for the sake of clarity.

What sort of form does the *Āryā* no. 12121212 have?

Gaṇa No.		Possible forms	remainder/ divisor			
a 1	12121212	÷ 4 =	3030303, 4.	Hence 1. Gaṇa has form	IV	= ⏑⏑⏑⏑
2	3030303	÷ 5 =	606060, 3	„ 2. „ „ „	III	= ⏑ — ⏑
3	(606060 + 1)	÷ 4 =	151515, 1	„ 3. „ „ „	I	= — —
4	(151515 + 1)	÷ 5 =	30303, 1	„ 4. „ „ „	I	= — —
5	(30303 + 1)	÷ 4 =	7576, 4	„ 5. „ „ „	IV	= ⏑⏑⏑⏑
6	7576	÷ 2 =	3788, 2	„ 6. „ „ „	II	= ⏑⏑⏑⏑
7	3788	÷ 4 =	947, 4	„ 7. „ „ „	IV	= ⏑⏑⏑⏑
b 1	947	÷ 4 =	236, 3	„ 1. „ „ „	III	= — ⏑⏑
2	(236 + 1)	÷ 5 =	47, 2	„ 2. „ „ „	II	= ⏑⏑ —
3	(47 + 1)	÷ 4 =	12, 4	„ 3. „ „ „	IV	= ⏑⏑⏑⏑
4	(12	÷ 5 =	2, 2	„ 4. „ „ „	II	= ⏑⏑ —
5	(2 + 1)	÷ 4 =	0, 3	„ 5. „ „ „	III	= — ⏑⏑
7	(10 + 1)	÷ 4 =	0, 1	„ 7. „ „ „	I	= — —

Hence the required form runs as follows:

⏑⏑⏑⏑ | ⏑ — ⏑ | — — | — — | ⏑⏑⏑⏑ | ⏑⏑⏑⏑ | ⏑⏑⏑⏑ | —
— ⏑⏑ | ⏑⏑ — | ⏑⏑⏑⏑ | ⏑⏑ — | — ⏑⏑ | ⏑ | — — | —

On this process, Hemacandra cites the following as his source:

naṣṭāṅke prathamaṃ bhakte gaṇāṅkaiś catur-ādikaiḥ |
śeṣa-saṃkhyo gaṇo deyo, labdhaṃ kuryāt sa-rūpakam || 1 ||[8]

punar bhajet, punar labdhaṃ sa-rūpaṃ; śeṣa-saṃkhyayā |
gaṇān dadyād gate śeṣe gāthāyāḥ prāk gaṇa-kramaḥ || 2 ||

6. *uddiṣṭe'ntyāl lād dvir, g: ekaṃ tyajet.*[9]

"In the given (form), beginniing with the last short syllable, (and proceeding towards the left), multiply by 2 (for each syllable); for a long syllable, deduct 1."

Since, in order to be able to multiply at all, one needs some number or other and since there is no reason to take a number larger than 1,[10] we should write down for the last L a 1, which yields 2 when multiplied by 2. For example, for the form — — u [which we obtained as an example of the *naṣṭa*] we have to proceed thus:

1.	L	$1 \times 2 = 2$
2.	G	$2 \times 2 - 1 = 3$
3.	G	$3 \times 2 - 1 = 5$,

i.e. this form has the serial number 5 in the tri-syllabic *prastāra*. [If the form is u — — — u, then we continue the process thus:

4.	G	$5 \times 2 - 1 = 9$
5.	L	$9 \times 2 = 18$

i.e. u — — — u is no. 18 in the *prastāra* of 5 syllables.]

For the *mātrā-vṛttas*, the rule is the following:

7. *ādyam antena hataṃ vy-adhastanam.*

"Multiply (the number of the possible forms of each *gaṇa*, starting from) the last by (the number obtained for) the preceding (*gaṇa*) and subtract (each time from the product, the number of) the remaining (forms after the actual form of the *gaṇa* concerned)."

For example, what is the serial number of the following form of *Āryā*?

sa jayati Kumārapālaḥ śrīmān avani-pati-śata-vinuta-caraṇaḥ |
nirmala-yaśasā dhavalita-bhuvanaś cakradhara-tulyaujāḥ ||

[This stanza has precisely the form which we obtained previously as the solution for the *naṣṭa* problem. This form should be kept in mind during the calculation that follows.]

Serial no. of the *guṇa*.	possible forms (*a*)	*Gaṇa* has the form no. (*b*)	remaining forms (*a-b*)		
b 8	1	1	—		
7	4	1	3	$1\times4-3=$	1
6	1	1	0	$1\times1-0=$	1
5	4	3	1	$1\times4-1=$	3
4	5	2	3	$3\times5-3=$	12
3	4	4	0	$12\times4-0=$	48
2	5	2	3	$48\times5-3=$	237
1	4	3	1	$237\times4-1=$	947
a 8	1	1	0	$947\times1-0=$	947
7	4	4	0	$947\times4-0=$	3788
6	2	2	0	$3788\times2-0=$	7576
5	4	4	0	$7576\times4-0=$	30304
4	5	1	4	$30304\times5-4=$	151516
3	4	1	3	$151516\times4-3=$	606061
2	5	3	2	$606061\times5-2=$	3030303
1	4	4	0	$3030303\times4-0=$	12121212

The given *Āryā* has the form no. 12121212 [that is to say, the solution to the *naṣṭa* problem has been found correct].

Hemacandra cites his source here also:

gaṇān uddiṣṭa-gāthāyāḥ saṃsthāpya tad-adho likhet ।
catuṣ-pañcādikaṃ saṃkhyāṃ sthāna-sthānocitāṃ tataḥ ॥ 1 ॥
hatvā hatvādyam antyena coparistha-gaṇād adhaḥ ।
pṛthag dhṛta-gaṇebhyo[11] 'tha gaṇa-saṃkhyāṃ vikalpayet ॥ 2 ॥
hatād dhāryocitā[12] *tāvad, yāvad ādyāṅka-saṃbhavaḥ* ।
tat-saṃkhyām uddiśed gāthām uddiṣṭa-pratyaye budhaḥ ॥ 3 ॥

[The different forms of the *prastāra* can be arranged into groups according to the following criteria: forms with only Gs; with just Ls; with 1 G and the rest Ls; with 1 L and the rest Gs; with 2,3,... Gs (or Ls) and the rest Ls (or Gs). The fourth *Pratyaya* teaches how to determine the numerical proportion of these groups for a desired number of syllables, without having to construct the entire *prastāra*.]

8. *varṇa-saman ekakān saikān upary upari kṣipet muktvāntyaṃ: sarvaikādi-ga-la-kriyā.*

"(Write down, one below the other,) as many unities as there are syllables and one more, and add always the lower one to the next one above, but excluding the last; (this yields) the graphic representation (of the form-groups) with just 1,2, etc. Gs or Ls."

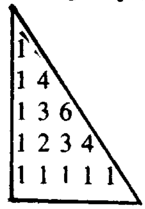

Fig. 6

[We begin, therefore, with 5 unities for 4 syllables (Fig. 6). Of these, the lowest is added to the next one above = 2. This 2 + the next 1 = 3; this 3 + the next 1 = 4; to the next 1 nothing must be added because it is the last number. Thus we get in the second column 1, 2, 3, 4. This column is treated again in the same manner and we get 1; 1+2=3; 3+3=6. Since 4 is the last number, nothing is added to it. Thus we get finally Fig. 6. The numbers forming the hypotenuse of the triangle show successively the number of forms thus:

4 G + 0 L	(1 form)
3 G + 1 L	(4 forms)
2 G + 2 L	(6 forms)
1 G + 3 L	(4 forms)
0 G + 4 L	(1 form).]

Hemacandra himself explains the process somewhat differently in his commentary. He performs only one addition in each column and in this manner construct the following number-columns:

1	1	1	1	1	1	1
1	1	1	4	4	4	4
1	1	3	3	3	6	6
1	2	2	2	3	3	4
1	1	1	1	1	1	1

Fig. 7

Of course, he achieves the correct result in this manner also, since the last column represents the required series. But his method does not quite agree with the text of the *sūtra*, or to be more precise, his method cannot fully be derived from the *sūtra*. For it is not clear from the *sūtra* why the 4 in the 6th column and the 6 in the 7th column (Fig. 7) are treated as *antya*, to which nothing may be added. Nor is it clear why unities are written once again at the top in the 4th and 5th columns, and 1 and 4 at the top of the 6th and 7th columns. Thus Hemacandra apparently modified the process, which I have described according to Kedāra, but he did not make it compatible enough with the *sūtra,* or he misunderstood his source.

In the case of the *artha-sama-vṛttas,* a number of forms must be excluded as in the *prastāra.* For instance, if we take the set of 4 syllables just discussed as the half verse of an *artha-sama-vṛtta,* then the form nos. 1,6,11,16 of the *prastāra* (see Fig. 2 above) must be excluded, and there will remain

3 G + 1 L (4 forms)
2 G + 2 L (4 forms)
1 G + 3 L (4 forms).

Likewise, a *viṣama-vṛtta* of 4 × 2 syllables is treated as a *sama-vṛtta* of 8 syllables, after excluding those cases which go against the *viṣama* principle. Thus the *sarvaikādi-ga-la-kriyā* for 8 syllables runs thus:

	8G	7G,1L	6G,2L	5G,3L	4G,4L	3G,5L	2G,6L	1G,7L	8L
sama-vṛtta	1	8	28	56	70	56	28	8	1
viṣama-vṛtta		8	24	56	64	56	24	8	

[In his commentary, Hemacandra just briefly states that the *sarvaikādi-ga-la-kriyā* for a *viṣama-vṛtta* having a set of 8 syllables is 8, 26, 53, 66, 53, 26, 8; but he does not tell us of a method with which one could, in theory, calculate how much is to be subtracted from each term of the *saṃa*-series. It is, however, clear that all the forms containing just Gs or Ls are excluded, and that all forms containing odd number of Gs and Ls can neither be *sama* nor *arthasama.* Thus the subtractions pertain only to the groups 6 + 2, 4 + 4, and 2 + 6. From the first and the last groups 4 forms each and from the middle group 6 forms have to be removed. Hence, Hemacandra's figures —— at least as they are reproduced in the edition available to me —— must be wrong.]

The *sarvaikādi-ga-la-kriyā* is extended also the *mātrā-vrttas.*

9. *ādya-bhedān adho 'dho nyasya parair hatvāgre kṣipet.*

"Write down the types of the first (*gaṇa*) one below that other, multiply them with those of the second (*gaṇa*) (according to the *kapātasandhi* principle), and add the resulting columns (and so on)."

From the commentary we learn the following method. The "types" of the first *gaṇa* are: no L (——, = 1), 2L (∪∪— and —∪∪, = 2) and 4L (∪∪∪∪, = 1). Write the numbers, 1, 2, 1, corresponding to these types in a column. In a second column next to it, write down, according to the manner of "closing the door leaves" (*kapāṭasandhikrameṇa,* i.e. just as the two leaves of the door overlap each other at the edge, when closed), the numbers of the second *gaṇa*, which are 1,3,1 (because there is an additional ∪—∪). The first column is multiplied by each numeral of the second, according to the *kapāṭasandhi* principle (see Fig. 8), and the three resulting columns are added together laterally. Next to this column of sums, write again the numbers, 1, 2, 1 for the third *gaṇa;* multiply and add as before. For the fourth *gaṇa* the column of multipliers is again 1, 3, 1, and so on. The calculations up to the fourth *gaṇa* are as follows (the commentary carries out this process only this far):

I	II			III				IV				
1	1			1	1			1	1			1
2	2	3		5	5	2		7	7	3		10
1 × 1 =	1	6	1	8	8	10	1	19	19	21	1	41
2		3	2	5	5	16	5	26	26	57	7	90
1			1	1 × 1 =	1	10	8	19	19	78	19	116
				2		2	5	7	7	57	26	90
				1			1	1 × 1 =	1	21	19	41
								3		3	7	10
								1			1	1

Fig. 8

[The column obtained last represents the groups of forms for 4 *gaṇas*. Since, in accordance with the formation of *gaṇas*, the number of L can only be an even number, it increases by 2 for each numeral of the column, that is to say, there 1 form with OL (where all the 4 *gaṇas* contain only Gs), 10 forms with 2L, 41 forms with 4L, 90 with 6L, 116 with 8L and so on up to 1 form with 16L (i.e. all Ls). This process is wanting in Nārāyaṇa's treatment, which is limited to *saṃkhyā, naṣṭa* and *uddiṣṭa* of the *Āryā.*]

Now the question is raised: how can we determine the number (*saṃkhyā*) of the possible forms of any arbitrary metre, without counting them in a table? There are two methods for this.

10. *te piṇḍitāḥ saṃkhyā.*

"The sum of these (numerals) is the number (of all possible forms)." Naturally one can determine the number of all possible forms for all metres by simply adding the group-numbers of the *sarvaikādi-ga-la-kriyā.*

The second method is shorter and more practical.

11. *varṇa-sama-dvika-hatiḥ samasya.*

"For the *sama-vṛttas,* (the number of possible forms is equal to) the power of 2 which equals the number of syllables (lit. is achieved by multiplying 2 by itself so many times as there are syllables)."

Examples given in the commentary.

Uktā	1 syllable	2 forms
Atyukta	2 syllable	$2 \times 2 = 4$ forms
Utkṛti	26 syllables	$2^{26} = 67108864$ forms.

12. *te dviguṇā dvi-hīnāḥ sarve.*

"This (number) multiplied by 2 and reduced by 2 gives the sum of all forms (of all metres from 1 syllable up to the desired number of syllables)."

The sum of all forms from *Uktā* up to *Utkṛti* is $= (2^{26} \times 2) - 2 = 134217726$; from *Uktā* up to *Gāyatrī* (6 syllables) is $= (2^{6} \times 2) - 2 = 126$.

13. *sama-kṛtī rāśy-ūnā artha-samasya.*

14. *tat-kṛtir viṣamasya.*

"(The number of the syllables in each *pāda* remaining the same, the number of possible forms) of the *artha-sama-vṛtta* is equal to the square of the *sama-vṛtta* forms minus their number, and of the *viṣama-vṛtta* is equal to the square of this (square minus the number of *sama* and *artha-sama* forms)."

For example, in a 4-syllabic *pāda*, the *sama-vṛtta-saṃkhyā* is $2^4 = 16$. As shown above, in the case of an *artha-sama*, the *prastāra* will go up 8 syllables, and thus the *saṃkhyā* is 2^8 or, to put it differently, $(2^4)^2$. From this, the *sama*-forms (2^4) have to be removed. Thus, for the *artha-sama-vṛtta* we get $(2^4)^2 - 2^4 = 16^2 - 16 = 256 - 16 = 240$. The *prastāra* must be extended up to 16 syllables for the *viṣama-vṛtta*. Hence, its *saṃkhyā* is $2^{16} = (16^2)^2$. From this, we should subtract the 16 *sama* forms and the 240 *artha-sama* forms. Hence we get $(16^2)^2 - 16 - 240 = 256^2 - 256 = 65280$.

15. *vikalpa-hatir mātrā-vṛttānām.*

"In the case of the *mātrā-vṛttas*, (the *saṃkhyā* is obtained by) multiplying (the number of) variations (of the individual *gaṇas*)."

This has already been shown in connection with the *prastāra* on the example of *Āryā*.

Now comes a general method for calculating the number of possible forms of a *gaṇa* having a specific number of morae. This is given in the following *sūtra*.

16. *ahkāntyopāntya-yogaḥ pare pare mātrāṇām.*

"(The numbers of the possible forms of the *gaṇas*) having (1;2,3...) morae[13] (constitute) a series in which each succeeding term is equal to the sum of the two preceding terms."

The commentary constructs the series up to the eighth term, i.e. up to the *gaṇa* having 8 morae.

No. of morae	1	2	3	4	5	6	7	8
			1+2	2+3	3+5	5+8	8+13	13+21
No. of possible forms	1	2	3	5	8	13	21	34

Finally, the sixth *Pratyaya* is dismissed briefly.

17 *dvi-ghnān-ekādhvayogaḥ.*

"The measure of space (required for writing down the *prastāra*) is double (the *saṃkhyā*) minus one."

A *sama-vṛtta* of 3 syllables has 8 forms. If we wish to write these in separate lines one below the other and count one aṅgula for each line and for each space between two lines, then we need $2 \times 8 - 1 = 15$ *aṅgulas* (i.e. 8 lines and 7 spaces inbetween).

After this, Hemacandra concludes with the remark: *iha ca prastāra-saṃkhyayor evopayogo; naṣṭādayas trayaś citramātram. adhvayogas tu puruṣecchānuvidhāyitvenāniyato, na ca kṣetraniyame phalam asti: iti nirupayogaḥ. pūrvācāryānusaraṇāt tv asmābhir uktaḥ.*

"Of these *Pratyayas*, *prastāra* and saṃkhyā have practical utility. *Naṣṭa, uddiṣṭa* and *sarvaikādi-ga-la-kriyā* are just curiosities.[14] *Adhvayoga*, however, cannot be determined exactly as it varies according to the fancy of the individual,[15] nor is any purpose served by determining such space. Thus is has no practical value. But we mention it all the same, in order to follow the ancient teachers."

(End of the work).

III. Piṅgala's Chandaḥśāstra

The verse in which Hemacandra enumerates the *Pratyayas* at the beginning of the chapter occurs also in Kedāra's *Vṛttaratnākara* (VI.1) with almost the same wording:

Prastāro naṣṭam uddiṣṭam eka-dvy-ādi-la-ga-kriyā |
saṃkhyānam adhvayogaś ca ṣaḍ ete pratyayāḥ smṛtāḥ ||

The textual variations are of no practical significance. It is impossible to find reasons for the priority of one version or the other. Therefore, the number and the sequence of the *Pratyayas* listed by Hemacandra is apparently the version commonly accepted in later times. On the other hand, the oldest exposition of the theory in Piṅgala's *Chandaḥśāstra* not only differs to a certain extent with regard to the sequence, but it also exhibits other factors which occasionally offer us an insight into the origin of the classical system. Piṅgala's commentator Halāyudha, however, takes the classical system as self-evident and reads it into Piṅgala's text.

Hemacandra's first three *Pratyayas* appear in Pingala's work also in the same sequence: *prastāra, naṣṭa* and *uddiṣṭa*. The following four *sūtras* deal with the *Pratyayas* (VIII. 20-23): *dvikau g-lau/ miśrau ca/ pṛthag lā miśrāḥ/ vasavas trikāḥ/*. According to Halāyudha's explanation correctly reproduced by Weber, their import is this. Write down a G, below it a L, below that once again the same, and a line of separation between the two for the sake of clarity, thus

—
u
—
u

In a second column next to this, 2 Gs and 2Ls are written and the line of separation is wiped off

—	—
u	—
—	u
u	u

Now this whole figure is repeated twice one below the other and separated by a line. In a third column next to this, 4 Gs and 4 Ls are written and the previous line of separation is wiped off. Thus we get exactly the *prastāra* for 3 syllables as shown in Fig. 1. There will be *vasavas trikāḥ*, "eight groups of three syllables," and these are the eight groups which are used in the scansion of all syllabic metres. Continuing the process in the same manner, we get 16 permutations of 4 syllables, 32 of 5, 64 of 6, and so on.

The basic principle of construction here is the same with which modern mathematics operates. It can be formulated somewhat in the following manner. For the first syllable, there are 2 possiblities G and L. Since both these can be followed again by G and L, there will be $2 \times 2 = 4$ possibilities for 2 syllables. Each of these 4 forms can be followed again by a G or a L with the result that there are $4 \times 2 = 8$ permutations for 3 syllables. Since all the possibilities for a specific number of syllables are thus connected always with either of the two possibilities of the next syllable, the number of permutations is double with each new syllable that is added. From this one can easily derive Hemacandra's rule for the *saṃkhyā* according to which the *saṃkhyā* is 2 raised to a power equal to the number of syllables.

We can see that Halāyudha's method is basically correct and leads almost exactly to Hemacandra's second method of *prastāra*. Hemacandra, however, puts everything into a single *sūtra*, thus somewhat obscurring the logic of the step-by-step construction. According to him, the first row is written all at once *yāvat* *saṃkhyaparımaṇam*. The method is right, but in order to derive it from Piṅgala's text, we have to twist the words and construe in a strange, even impossible, manner.

Weber translates the *sūtra, miśrau ca* thus: "and [a G and a L are to be] mixed together (likewise twice each)." This expression for the arrangement of the second column sounds no less strange in German than in Sanskrit. But it is already an attempt by Weber to replace Halāyudha's interpretation with something more plausible. Halāyudha, in fact, explains: *gakāro gakāreṇa saṃśliṣṭo miśra ucyate, lakāraś ca lakāreṇa; miśrāv iti gakāra-lakārābhyāṃ pratyekam abhi saṃbadhyate* ..... Accordingly, *miśra* should have the meaning, as Weber rightly noted, of *amiśra* here! Equally unconvincing is the explanation of the next *sūtra: pṛthag-iti vijātīyāsaṃsargam āha. tena prathamāyām āvṛttau na lakāra-praveśaḥ, dvitīyāyāṃ na gakārasya*. But *pṛthag* does mean "singly" and not "exclusively".

At all events, this *sūtra* should read *pṛthag g-lā miśrāḥ*. Weber surmises this reading[16] rather hesitatingly, after having stated previously: "There is some difficulty here in the *lāḥ* of the text. Apparently the same word cannot denote short syllable, but only syllable. In fact, short syllable is not called *la* but just *l* (cf. *lam* 4.53, 48, 50). But this general meaning 'syllable' for *la* cannot be attested otherwise" (p. 428, n. 2). Regarding rule 23 b of Piṅgala, Weber even says: "probably just the number of combinations is meant by 'the number of *la*' .... However, *la* in this sense is as rare as in the meaning 'syllable' which is required in rule 22." *First of all, the distinction made here between l* and *la* is baseless. Later writers on prosody employ consonants with or without vowels, according to their fancy or need, as abbreviations for G and L and as designations of the eight *gaṇas* (*m, n, s.* etc.). In his definitions, Piṅgala employs them as far as possible without a vowel. Usually he combines each two in dual with – *au* (*bhrau, ntau*, etc.). If their number is odd, the last abbreviation after one or more pairs is without a vowel. From the dual ending in – *au*, one cannot see whether it is a-stem or consonant-stem, but where a plural is needed, Piṅgala forms it with *āḥ* and not *aḥ*. In 5.30: *Śuddhavirāḍ ṛṣabhaṃ ta-j-rāḥ*, we can see at the same time how *ta* is used instead of *t* either for the sake of metre or because there are too many consonants. See also 6.37: *Bhujaṅgaprayātaṃ yāḥ* and 6.38: *Sragviṇī rāḥ*. Hence, Weber's proposal to emend 8.22 as *glo* is unnecessary. Finally, it would be sufficient just to refer to 5.27: *vipulā yug laḥ saptamaḥ*, which Weber himself translated as "when in the same *pāda* (2,4) the seventh syllable is short."

Confusion cannot arise from the juxtaposition of *l* and *la*, for the assumption that *l* or *la* can denote such disparate things as "short syllable", "syllable", or even "combination" should never have been made in view of the ingenious exactness of Indian terminology. *l* or *la* can never be anything but *laghu*, "light", i.e. metrically short syllable and this sense, as will be shown below, fits in Weber's rule 23b as well.

I shall not discuss Halāyudha's interpretation of the *prastāra sūtras* in further detail. Instead, I shall attempt to show how another, quite contrary, interpretation can, in one go, remove all the contradictions mentioned above. In Piṅgala's *sūtra, dvika* =

2; therefore, *dvikau* = 2 × 2; *dvikau g-lau* = *dvikau gau* + *dvikau lau* = 2 × 2 G + 2 × 2 L, and this is exactly what is written in the third column according to Halāyudha. To this are added two *"miśrau" dvikau*, i.e. 2 groups of 4 which are mixed in pairs of G and L. That is to say, 2G, 2L, 2G, 2L, and this corresponds to Halāyudha's second column. Finally, we add *pṛthag g-lā miśrāḥ*, "G and L mixed singly," i.e. always one G and one L alternately. This is Halāyudha's first column. Of course, in this reverse construction, it is not possible to continue this table in a like manner for variations of 4, 5, or more syllables, and this fact can be explained in three ways.

1) The addition of the concluding *sūtra: vasavas trikāḥ*, the significance of which was noticed by Halāyudha as his remark *vispaṣṭārtham idaṃ sūtraṃ prastārād eva saṃkhyā-paricchitteḥ* shows. Only when the process is concluded at this stage, i.e. when the process is limited to the arrangement of the 8 *trikas*, i.e. *gaṇas*, does the addition of the statement here make sense, so does the absence of a similar statement, e.g. in the case of the 4 variations with 2 syllables.

2) The fact that a contrary interpretation was at all attempted by Halāyudha shows that it was felt necessary to find a method which could be continued *ad infinitum*. Halāyudha refers to such a continuation in his commentary, while the very *sūtras* for this are wanting.[17]

3) Likewise the interpolation of the two rules which Weber designates as 23 a-b: *ekottara-kramaśaḥ; pūrvapṛktā la-saṃkhyā*. He translates them as follows: "In the series each time by one more; the number of *la* is combined each time with the previous (number)." Here the expression *"la"* with its supposed meaning "combination" did not satisfy Weber himself and, as has been stated above, it is quite impossible to render *"la"* as "combination". Obviously the (interpolated) *sūtra* is not aptly formulated, but its import can only be this: for each extra syllable, add to the existing scheme so many (Gs and) Ls as the number (of lines). That is to say, next to the entire old column, write once all Gs and then a second time all Ls. For this "addition" of Gs and Ls, the expression *pṛktā*, to which Weber took objection, is quite suitable.

This is all Piṅgala and Halāyudha offer on the *prastāra*. Here we miss, therefore, not only a discussion of the *artha-sama-vṛttas* and *viṣama-vṛttas* (which is mathematically uninteresting) but also another method for *prastāra* which Hemacandra treats as the second and Kedāra as the only method. Consequently, it seems that this method was discovered only later, and presumably in the following manner: keeping the *prastāra* table in front of one's eyes, an attempt was made to derive each line successively one after the other. While doing so, it was discovered that the formation of each line from the preceding line can be summed up in a single simple formula: each time the first G becomes L, all the following syllables remain the same but the preceding ones become G. That this process is also mathematically significant in so far as it ensures the inclusion of all the possibilities is evident already from the very fact that this method leads directly to the table of the first method from which, after all, it is derived. But we can also offer a direct reasoning for it.

If we, take, for instance, the *prastāra* for 4 syllables shown in Fig. 2 above, then first of all the first syllable is converted to L. Its possibilities are exhausted with this, and hence now the second syllable must be changed into L. While doing so, however, we begin the process for the first syllable once more from the beginning (with G) and thus combine with the second possibility of the second syllable first the first possibility and then the second possibility of the first syllable. Thus the possibilities for the first two syllables are exhausted, and hence we should now take up the third syllable and change it into L. But in order to combine with this second possibility of the third syllable all the possibilities of the preceding syllables, we must begin from the beginning once again for these two (with 2G) and repeat with them the existing method until we obtain only Ls for the first three syllables.

As we can see (and as the table in its totality shows), the whole process with its continuous priority of G over L implies that for each syllable-number the available possibilities are exhausted as soon as we reach all Ls, just as in the case of the first syllable, the L is the (second and the) last possibility. But always a new syllable is changed only when it is the "first G," i.e. when it is preceded by just Ls and all the possibilities for the preceding syllables are exhausted. But if a new syllable is changed, i.e. when the second of its two possibilities is chosen, then the process begins for all the preceding syllables once again the beginning, so that all the possibilities of the preceding syllables are inevitably combined with the second possibility of the next syllable.

Hemacandra's and, more particularly, Kedāra's preference for this process can be explained not only from the fact that it is more convenient in so far as it allows us, even when the number of syllables is large, to write one horizontal line after the other without much thinking; but the weightier reason should have been rather the fact that this process is not so transparent in its mathematical reasoning that it appears all the more amazing. The more remarkable and, at first sight, the more inexplicable a method is the prouder its inventor — this can be seen still more clearly in a whole series of *Pratyaya* feats. In this respect, the primitive scientific thinking is quite the opposite of ours. While we take pride in clear intelligible derivations and logical and provable reasoning, Indians seek to astonish, as far as possible, through strange and apparently inexplicable feats, the explanation and proof of which is left, without exception, to the sharp intellect of the reader.

The preceding discussion should have shown how extra-ordinarily ingenious and mathematically productive the construction-system of *parastāra* is. This will be evident more clearly when we observe the two methods, *naṣṭa* and *uddiṣṭa;* their simplicity would be marred if we change the construction of the table in even a minor detail.[18]

Piṅgala's treatment of *naṣṭa* and *uddiṣṭa* is exactly like Hemacandra's. *Naṣṭa* is explained correctly by Weber; in the case of *uddiṣṭa* too he understands the facts correctly with a mathematicians's help but, as he says, much remained obscure to him in Halāyuddha's commentary. Hence, before I discuss the mathematical principles underlying both the methods — Weber touches upon then briefly in connection with *nasta* only — the obscure points must be explained first.

Halāyudha states ... *tatas tasyānte yo lakāraḥ sajātīyāpekṣayā tam ādau kṛtvā prātilomyena dvir āvartayet. tatra nirākārāyā āvṛter asambhavāt prathamātikrame kāraṇābhāvād eka-saṃkhyā labhyate....* Weber translates this in the following manner: "Then one should place the last short syllable at the beginning (?) of similar syllables[19] and repeat twice (double it) backwards. Since there can be no repetition[20] of that which has no external form (?), we obtain the number one in so far as there is no reason for passing over for the first time(?)."

The first sentence is wrongly broken up by Weber. If we put a comma after *sajātīyāpekṣayā* and not before it — this is indicated syntactically even by *yo ... tam* — then it can be translated thus: "Thereupon one should take as the beginning the short syllable that occurs at its (i.e. of the form) end in relation to similar ones," i.e. the short syllable which occupies the last place — last place not in an absolute sense (because the *pāda* can end in one or several Gs) but in relation to similar syllables; in other words, simply the last of the short syllables.

The second sentence is quite clear; Hemacandra exactly repeats the part which supplies the rationale (cf. p. 25 above). "Thus we get the numeral 1 since a repetition without an object cannot take place and, on the other hand, since there is no reason to pass over the 1 (*prathama* is the first numeral and not for the first time)." What Weber surmises in the footnote is thus correct; here we are given a justification "for beginning the calculation with 1": if you want to double, there must invariably be some number to be doubled; but there is no reason to take a larger number than 1. For our feelings, this justification appears rather clumsy and indeed superfluous, but it is very typical of the primitive mode of thinking.

Both methods — of these the *uddiṣṭa* is in fact the mirror image of the *naṣṭa* — are based on the peculiar nature of the *prastāra* table. Weber already pointed to this fact briefly in connection with the *naṣṭa* with the remark "that at the beginning of each individual line of the *prastāra,* there occur alternately one G each at the unequal (odd) place and one L each at the equal (even) place" (p. 440). Likewise, in the second (or 3rd, 4th, etc.) column also, there will be alternate pairs (or groups of 4, or 8, etc.), of which the odd groups contain only G and the even groups only L.

If we want, for instance, to determine the 13th form in the *prastāra* of 4 syllables, the odd number 13 shows that the first place is occupied by a G. If we take out from the table all the lines beginning with G and write them down separately, it will be a new table reduced to half the original (Fig. 9). Here, instead of alternate pairs of G and L in the second column, we have single G and L but in the same order as they were in the first column of the old table (Fig. 2).

—	—	—	—
—	U	—	—
—	—	U	—
—	U	U	—
—	—	—	U
—	U	—	U
—	—	U	U
—	U	U	U

Fig. 9

Now in order to determine which line is occupied by the form in question, we should first find out to which pair of lines in the old table it belongs. Since each odd number makes a pair with the next higher number, this is found by dividing the next higher number by 2. To put it differently: add 1 before division. This addition is, of course, unnecessary in the case of an even number. In our example, form 13 occupies $(13+1) \div 2 = 7$th line in the table. This odd number 7 indicates a G in the second place, and so on and so forth.

In the case of the *uddiṣṭa,* first of all the beginning with the last L instead of the very last syllable is just an abridgement of the process, but it obscures its logical construction. We can apply this process to any number of final Gs also without any change. Since we begin with 1, and in the case of G we double the 1 and subtract 1 from the product, the result remains 1 for any number of Gs until we reach a L (just as the reverse holds good in the *naṣṭa;* as soon as we reach 1, then there will occur only Gs).

Now, *uddiṣṭa* is based on the principle that the possibilities for the position of a given form are halved at each syllable, starting from the last syllable and proceeding towards left, until finally there remains only one possibility.

In each *Prastāra* table, the last column consists only of 2 groups. Of these, the first group contains only Gs and the second only Ls (see Fig. 2). Hence the quantity of the last syllable determines to which of the two halves of the table (i.e. the first or the second) the given form belongs. In the same manner, the penultimate syllable determines one of the two quarters within the half (that has already been determined), and the third syllable from the last determines one of the two 1/8th parts within the quarter previously determined, to which the given form belongs. Finally, in the case of the first syllable, both the fractions represents single lines (e.g. in the case of 4 syllables, there are 16 forms and 1/16th part of the table is a single line). Hence, by selecting one of them, the number sought is arrived at. The continuous multiplication by 2 is based on the fact that there are twice as many quarters as there are halves, twice as many 1/8th parts as 1/4th parts, and so on. Subtraction of 1 takes place because each G refers to the first of the two available possibilities.

As in the case of *prastāra,* there is also a second improved method for *uddiṣṭa.* Hemacandra does not mention it, but Kedāra teaches it as the only method (6.5, cf. Weber, pp. 431, 443). Starting from the left, we should write 1, 2, 4, 8, 16 ... above the syllables. If we add the numerals written above the short syllables plus 1, the sum is the required number.[21] This looks amazing but is based on a very simple principle. For instance, one look at the *prastāra* table for 4 syllables (Fig. 2) will show that a form with a final L (above which will be written the numeral 8 because it is the 4th syllable) is preceded by at least 8 forms with final G. If there is a L at the third position (over which is written 4), then a form with a final G is preceded by at least 4 forms, and a form with a final L by at least 8 + 4 = 12 forms, and so on. Thus the decisive factors are the Ls, and therefore only the numerals written above them are added together. Those above Gs are written just for the sake of uniform progression. Since the addition of these numerals gives only the sum of the preceding lines, the serial number of the given form is higher by 1. Hence, we should always add 1 at the end.

The next two *Pratyayas, sarvaikādi-ga-la-kriyā* and *saṃkhyā* are treated by Piṅgala in the reverse order. For the time being, I shall follow this order; its probable reasons and the conclusions that can be drawn from it will be discussed only when I take up the *sarvaikādi-ga-la-kriyā.*

For our way of thinking, the *saṃkhyā* appears to be most obvious of all *Pratyayas* and a necessary corollary of the *prastāra,* much more than the *naṣṭa* and *uddiṣṭa.* Hemecandra's rule *varṇa-sama-dvika-hatiḥ* has been explained already on p. 29 above. Instead of this brief formula which reduces the solution to the simplest mathematical expression,[22] Piṅgala teaches a method which, at first sight, appears to be strange and complicated, but in reality, is very ingenious (cf. Weber's detailed discussion, p. 444 ff.[23]). It is based, as Weber already explained, "on a very ingenious way of simplification of the exponent" which results in the reduction of the operations needed for calculating the *saṃkhyā.* For instance, in order to calculate the value of 2^{11}, we have to multiply 2 ten times with 2. But we can go about it in another way. The modern mathematician will factorize 2^{11}, thus:

$$2^{11} = 2^{10}.2 = (2^5)^2.2 = (2^4.2)^2.2 = [(2^2)^2.2]^2.2.$$

To calculate the last form, we need just to perform thrice squaring and twice multiplication, i.e. 5 operations instead of 10. The indian method leads exactly to this kind of simplification of higher powers into lower with an exponent not larger than 2. To know when the squaring is done and when the multiplication with 2, the exponent (11 in our example) is halved continuously; if we reach an odd number, it is reduced by 1. Where an even number is halved, we have to square later and this is marked with an index number '2'. Where 1 is subtracted, we have to perform the multiplication late with 2, and this is marked with the index number 'zero'.[24] Thus we write

$11 - 1 = 10$('0'); $10 \div 2 = 5$('2'); $5 - 1 = 4$('0'); $4 \div 2 = 2$('2'); $2 \div 2 = 1$('2'). The calculation must, of course, begin with the last index number. This becomes clear, if we put each index number below the power from which it was derived, in the following manner:

2^{11}	2^{10}	2^{5}	2^{4}	2^{2}
'0'	'2'	'0'	'2'	'2'

The identity between the Indian method and the modern method will immediately be evident if we write down the Indian index numbers in the order of calculation, below the final form as given above:

$$2^{11} = [(2^2)^2.2]^2.2$$
$$2\ 2\ 0\ 2\ 0$$

This method is conceived in a highly ingenious manner. The only problem is why do we find the more primitive simple rule in such a late work as Hemacandra's. For, I myself would not be able to point out to a way by which one could reach Piṅgala's process without first knowing Hemacandra's rule. Piṅgala's method can represent only the successful attempt to simplify the practical calculation. Hence we are left only with the hypothesis that Hemacandra gave preference to the strictly mathematical formula in its stark simplicity — just for the sake of brevity if not for anything else — over the method which is only relevant to the practice and which obscures the mathematical principle.[25].

While the *Agnipurāṇa* follows Piṅgala, Kedāra felt it necessary to replace Piṅgala's method with simpler process which represents an apt corollary to his *uddiṣṭa* method. According to him, the *saṃkhyā* is obtained by writing once again the numerals 1,2,4,8.... above the syllables of the metre and adding them — this time all of them — and an extra 1. This becomes clear when the remember that in *uddiṣṭa,* the addition of all numerals, according to Kedāra's rule, has to take place always at the last line consisting of all Ls (and their sum is the number of the proceeding lines). When we add 1 to this (i.e. the last line), we get the number of all the forms.

The *saṃkhyās* of the metres having 1,2,3,4,5,... n syllables are — it must have become clear from what has been said above — $2^1, 2^2, 2^3, 2^4, 2^5, \ldots 2^n$. This is a geometrical series in which the first term and the common ratio are 2. Now for the sum of this series, there is a formula in Piṅgala's work as well as in Hemacandra's: the sum of the first n terms is equal to twice the n-th term minus 2.

Indians may have discovered this formula also in an empirical way; at least, I would not know a simple theoretical way for this. In order to complete the picture, it must be added that the formula can of course be derived easily from the general formula for the summation of geometrical series. It is well known that this formula is

$$S = a. \frac{e^n - n}{e - n}$$

where S is the sum up to the *n*-th term, *a* the first term and *e* the common ratio. For our example, substituting 2 for *a* and also *e*,

we get $S = 2. \frac{2^n - 1}{2 - 1} = 2 \,.\, 2^n - 2.$

Since, according to Hemacandra's rule for the *saṃkhyā*, the *n*-th term is 2^n, this is exactly the Indian formula.

In Hemacandra's work, the summation formula is followed by the *saṃkhyā* for the *artha-sama* and *viṣama-vṛttas*. Piṅgala has a counterpart for this but, strangely enough, not in the concluding *Pratyaya* chapter where it should properly belong to, but at the beginning of the 5th chapter in connection with the definition of the *sama, arthasama* and *viṣama*. Therefore, Kedāra too does not say anything of this nature in his *Pratyaya* chapter, and not even where he introduces the concepts of the *sama, artha-sama* and *viṣama*. According to Weber (p. 416), the whole *Pratyaya* chapter of Piṅgala is "purely a late appendix just to these last passages [i.e. 4.53 and 5.3-5]; while the possible combinations for *artha-sama* and *viṣama* metres are treated in these passages, in the appendix the calculations for the *sama-vṛttas* are given as a special case." But in view of the situation in Kedāra and of the entire history of the development of the *Pratyaya* theory, it is quite certain that the speculations about the *saṃkhyā* of the *artha-sama* and *viṣama* represent, on the contrary, a later development of a system, which is available to us at the end of Piṅgala's work in its older form, without this development. Of course, the final chapter of Piṅgala's *Chandaḥśāstra* itself may have been a late addition and considerably younger than the original work.

From the mathematical point of view, this section on the *artha-sama* and *viṣama* is not of further interest. From what we have said, everything must have become sufficiently clear. The only problem is about the *rāśī* in the expression *rāśy-ūna* which Hemacandra borrowed from Piṅgala. Weber understands it as "square-root" and explains it as (p. 36n) "*rāśi,* quantity, here obviously means *mūlarāsi.*" In fact, Halāyudha also explains the word as *mūlarāśi.* On the other hand, Hemacandra says: *sā rāśy-ūnā samavṛtta-saṃkhyā-hīnā,* and thus seems to understand it simply as numericl quantity. The problem has some significance only in so far as Weber would like to consider the passage as one of the oldest testimonies for the knowledge of square-root. Objectively, it is however quite possible that *rāśi* here has simply the old meaning of "quantity".

Modern mathematics distinguishes between three types of complexes. From a given number of elements which can all be different or partly the same, if we change the order of all possible types, then we have permutations. Combinations, on the other hand, are arranging of n elements into groups of p each (p-th class), but in such a manner that the resulting groups do not differ from each other in the matter of sequence. If this proviso is dropped, i.e. permutations are also made within each single combination, we get variations of n elements of the p-th class. Thus these are combinations along with permutations possible for each combination. According as there are several similar elements are not, we speak of combinations and variations with or without repetition,[26] but we may add straightaway that in the case of the *Pratyayas*, both take place with repetition only, since the number of class (i.e. here the number of syllables) can always be greater than the number of elements which remain constantly 2 for ever (G and L).

Although the concepts just defined are not presented as such and are not designated with special names in the *Pratyaya* theory, the *sarvaikādi-ga-la-kriyā* to be discussed now shows that Indian were practically led to these concepts and actually operated with them without being aware of them.

According to the definitions given above, it is clear that so far we have been dealing exclusively with variations of 2 elements. On the other hand, the individual groups of the *sarvaikādi-ga-la-kriyā* offer us combinations of 2 elements, and the numerals in each row indicate how many permutations are possible for each combination; in other words, the *sarvaikādi-ga-la-kriyā* splits up, to a certain extent, the variations of the *prastāra* into combinations and permutations connected with the combinations.

Before we discuss the mathematical significance of this *Pratyaya* and further, we must first ascertain the factual position in Piṅgala which Weber totally misunderstood.

According to Weber, the last -sūtra of the work reads thus: *pare pūrṇam pare pūrṇam iti*. This itself is wrong. In reality, they are two *sūtras* with the same wording. Weber's remark, "According to the enumeration of the *sūtras* given in R at the end, *sūtra* 18 ought to contain 17 *sūtras*, but there are only 16," is without foundation; so also his observation that "that our author [Piṅgala] may have had in mind something like the *meru-prastāra* [i.e. *sarvaikādi-ga-la-kriyā*] does not follow from his words in any way" (p. 455). Now it is possible to argue that the repetition of the last *sūtra* originally indicated merely the conclusion of the work and that the second *pare pūrṇam* was given a different sense at a later time. But this assumption is opposed by the fact that *sūtras* are not repeated at the end of other chapters and that the conclusion is indicated by just *iti*. Therefore, we may have to assume that the end (which itself might be an addition) of the whole work is indicated by *sūtra*-repetition. But it is more likely — note what has been said above on the history of the *prastāra* — that the *sarvaikādi-ga-la-kriyā* is indeed a later addition to the *Pratyaya* system and that, in order to incorporate it, the last *sūtra* was repeated and was interpreted differently. That the *sarvaikādi-ga-la-kriyā* is a later addition is intrinsically probably not only from its mathematical character but it also explains the different sequence of the *Pratyayas* in

Piṅgala's text as shown above. Be that as it may, the text available to us as it is printed in the Kāvyamālā edition contains two sūtras — 33: *pare pūrṇam,* 34: *pare pūrṇam iti.*

Weber understood the first *pare pūrṇam* correctly. Coming after the formula for the summation of the *saṃkhyā* is the double of the saṃkhyā for the preceding number which is less by 1. Thus this *sūtra* offers a certain substitute for Hemacandra's direct rule for the *saṃkhyā*, instead of which, as we have seen, Piṅgala gave a peculiar method of calculation.

As against this, the second *pare pūrṇam* teaches the construction of the so-called *meru-prastāra,* i.e. a number triangle made up of rectangular cells *(caturasra-koṣṭha)*. On this Weber observes at p. 453f: ".... whose summit consists of a square, under which there occur 2, 3, 4, 5, ... squares successively. In the topmost square we should write the numeral 1. In the two squares of the second row, we write first in each the total number of combinations listed in the third row. In the three squares of the third row, in the middle one the double, and in the two lateral squares the simple number of the combinations of the fourth row are to be written. Likewise in the fourth row, in each of two middle squares the double and in each of the two lateral squares the simple sum of the combinations of the fifth row, and so on."

I do not think that Weber himself made an attempt to construct a number triangle according to these statements. Otherwise, he would invariably have noticed that his interpretation contains total contradiction in itself. According to the words with which he proceeds, there should occur, for instance, in the third row the 4 combinations of 2 syllables, in the fourth row the 8 combinations of 3 syllables etc. Accordingly, the three squares of the third row are to be filled with 8, 16, 8, that is to say, 32 "combinations," whereas there ought to be only 4. From Weber's wording, it is , however, apparent that he does not see any connection between the number triangle on the one hand and the formation of the "combinations" which visually represent the *sarvaikādi-ga-la-kriyā* on the other, for he goes on to say: "*thereafter, however,* (emphasis added) in the second row, the double quantities of a syllable —, ᴗ are to be written. In the third row there will be the four combinations of 2 syllables viz. in the first square — —, in the third ᴗ ᴗ, in the middle square the same with a short syllable (*dve ekalaghunī*). The fourth row should contain the 8 combinations of three syllables ..." etc.

It does not need reiteration that Weber simply misunderstood the relation between the two sections of Halāyudha's commentary. In reality, the first section explains the method of constructing the number triangle and then second section explains the significance of each individual number. The number triangle is constructed according to the following simple principle: in each square of the immediately following row (*pare*) comes the sum (*pūrṇam*) of the numbers written in the two square above it. In the topmost square is a 1; in the square above which there is only one square, we write the number which is in the single square above, that is to say, in the two squares of the second row, and in the first and the last of all other rows, we write just 1. Thus we obtain the following triangle.

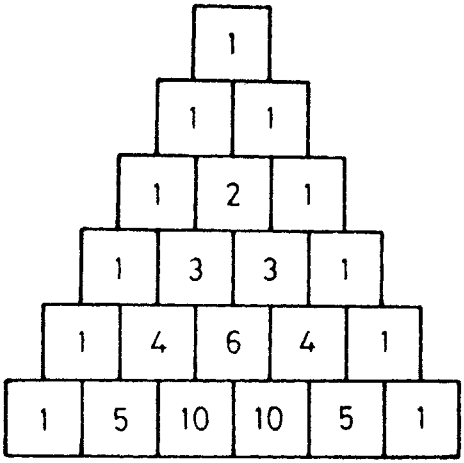

Fig. 9 (a)

This triangle is exactly the same as the one shown in Fig. 6, and Weber constructed such a one correctly according to Kedāra's instructions. Weber errs, therefore, when he compares Kedāra's "theoretial calculations" as "far superior" to Piṅgala's "purely practical method" (p. 445). But what he does not seem to have noticed is that this triangle is nothing but the famous *Pascal's triangle,* also called *tabula arithmetica* or *Wundertafel.* If we read the details about Pascal's work on the "tirangle arithmétique" which appeared in 1654,[27] the agreement with Halāyudha is really astonishing. Just like him, Pascal also first constructs the triangle just mechanically and then teaches its use, i.e. the significance of the numbers in the triangle. Just as Halāyudha constructs the triangle with *caturasra koṣṭhas,* so does Pascal with "cells" the purely external difference being that his triangle seems to have been turned by 45 degress so that it has the position as shown in Fig. 10. If we keep this fact in mind, then Pascal's rule for filling up the cells, viz. "the number of each cell is equal to the same in the cell which precedes in the perpendicular row plus the same in the cell which precedes in the parallel row," is exactly indentical with the *sūtra pare pūrṇam,* as Halāyudha explains it. Finally, Pascal's mathematical treatment of this triangle is no doubt more versatile and exhaustive than that of the Indians, but the second purpose taught by him, viz. "usage of the tirangle for combinations," corresponds exactly to the mathematical significance of the *sarvaikādi-ga-la-kriyā.*

1	1	1	1	1	1	1
1	2	3	4	5	6	
1	3	6	10	15		
1	4	10	20			
1	5	15				
1	6					
1						

Fig. 10

The mathematical principle underlying the *meru-prastāra* can be summed up as follows. The numbers of permutations of each single combination of 2 elements with repetition of n-th class constitute the binomial series for the n-th power, or conversely, the individual terms of the binomal series for the n-th power show, at the same time, the number of permutations of each individual combination with repetition of 2 elements of the n-th class.

That this is so becomes evident from the derivation of the binomial theorem. But it will lead us too far away to present it at this place; likewise the mathematical problems of the *sarvaikādi-ga-la-kiryā* and their solution can be shown here only briefly, with the request that those interested in further details may look them up in any text book on mathematics.

Piṅgala's method differs only superficially from Kedāra's or Hemacandra's. One has just to rearrange Fig. 6 in such a way that the hypotenuse becomes the base of the triangle to convince oneself that exactly the same additions are performed according to Kedāra as with Piṅgala. Thus both processes are based on a law — in India in any case it was discovered empirically — which may be fomulated thus: in the number triangle as well as in Pascal's triangle each number is equal to the sum of the two numbers above it. The underlying mathematical connection is expressed by the following formula in modern mathematics:

$$n_p + n_{p+1} = (n+p)_{p+1}.$$

Its derivation and proof would lead us too far away; interested readers may consult on both points any text book on elementary analysis. One example, however, may be given here to make the thing clear.

The binomial coefficients of the 4th and 5th power are

	1		4		6		4		1	
1		5		10		10		5		1

If we rewrite each coefficient in the form of a factorial that has been divided and at the same time the base 4 + 1 instead of 5, we get

$$\begin{array}{ccccccccccc} & 1 & & 4_1 & & 4_2 & & 4_3 & & 4_4 & \\ 1 & & (4+1)_1 & & (4+1)_2 & & (4+1)_3 & & (4+1)_4 & & (4+1)_5 \end{array}$$

Now, according to the above formula, $4_1 + 4_2 = (4+1)_2$; or $4_2 + 4_3 = (4+1)_3$ etc. Thus the law formulated at first is shown with specific numbers.

Adhvayoga, the sixth *Pratyaya,* represents, as has been stated above, one of those "unproductive devious paths," both from the mathematical and practical points of view. Indians also recognised this fact quite early, for it is actually discussed only by Hemacandra, Kedāra and Rątnākaraśānti (*Chandoratnākara,* ed. Huth, p. 19, v. 18)[29] Hemacandra states that he includes it only because the ancient teachers had done so. Accordingly, one would expect to find it, above all, in Piṅgala's work, but remarkably enough this is precisely not the case; instead, we find it only in a strange interpolated passage discussed by Weber at length on p. 432ff. Halāyuudha's commentary has nothing on this, but he himself probably knew about this *Pratyaya,* as can be seen from his remark, partly cited above: *ṣaṣṭha-pratyayo 'py adhvaparicchittirity eke; so' ty alpatvāt* ... Nārāyaṇa also quotes this remark (with the variant *alpaphalatvāt*). Bharata's *Nāṭyaśāstra,* to be discussed in the next section, as well as the *Prākṛta-paiṅgala* do not mention *adhvayoga* at all. It is difficult to decide at present whether this primitive-looking *Pratyaya* is a remnant from quited old times but found no acceptance even in Piṅgala's system, or whether it is a later addition that found no response and was given up soon after. Yet, especially the fact that Hemacandra feels obliged to discuss it and even those who are not interested in it know it all the same as a part of the old system suggests that the first assumption is more probable.

IV. Bharata's Nāṭyaśāstra

The *Pratyaya* theory is such a well recognised constituent of the metrical system that it is discussed not only by the acutal writers on prosody,[30] but it finds a place in such works also which treat prosody *en passant* as one of many topics. Thus we find it — aside from the *Agnipurāṇa* to which Weber refers throughout his study — in the *Nāṭyaśāsta* of Bharata as well. It is well known that this immensely valuable text is in such a deplorable condition that there is no complete critical edition available, although a beginning was made at two places.[31] In particular, the section on the *Pratyayas* suffered considerably; perhaps the not so simple subject matter may have been responsible for this. The passages cited from here by Regnaud in his monograph on Bharata's prosody[32] are, to a large extent, a totally incomprehensible chaos that cannot be disentangled; his translation — in parts designated prudently as paraphrase and for some stanzas not given at all — deals quite often with things that do not really occur in the text.[33] On the basis of the foregoing discussion, I think it is now possible to bring about some order at least into the text. Therefore, I give below the short section on the *Pratyayas* as a minor contribution to Bharata's text.

Besides Regnaud's text (R), I used the edition of Batuk Nath Sharma and Baldev Upadhyaya[34] (Be) and the Kāvyamālā edition[35] (K). R and Be contain the (older) recension B of the *Nāṭyaśāstra*[36] while K contains the variant recension A. In the latter, the first part of the section is wanting and the XIV chapter begins with verse 58b of R. The first verse of our section is R 51 = Be XV. 101b-102a.

The sequence of the verses differs from A to B, and it is not even remotely correct in either version. Because of the almost unsurmountable confusion reigning here, every attempt so far at reconstruction, let alone translation, had to fail. Moreover, the introductory verse and those on the *prastāra,* which still occupy the correct place and are available in correct sequence, are followed by some stanzas which definitely do not belong here; even their contents show that these are probably later additions. My text which makes use of both recensions in an eclectic manner is still far removed from the ideal text which can perhaps be never achieved in the case of Bharata. Yet I think that my sequence of stanzas may be regarded as final. Instead of a concordance of both recensions, I give after each verse its serial number according to Regnaud and then the same according to the Kāvyamālā edition.[37]

evaṃ tu chandasām eṣāṃ prastāra-vidhi-saṃśrayam |
lakṣaṇaṃ sampravakṣyāmi, naṣṭoddiṣṭaṃ tathaiva ca || 51

[*prastāro 'kṣara-nirdiṣṭaḥ, sa mātroktas tathaiva ca |*
"dvikau glāv" iti varṇauktau (!) "mandrāv" ity api mātrikā ||] [52]

Ia *guror adhastād ādyasya prastāre laghu vinyaset |*
agratas tu samo, deyā guravaḥ pṛṣṭatas tathā || 53

prathamaṃ gurubhir varṇair, laghubhis tv avasāna-jam |
vṛttaṃ tu sarva-chandassu: prastāra-vidhir eṣa tu || 54

Ib *gurv, adhastāl laghu nyasya tato dvir dvir yathoditam |*
nyaset: prastāra-mārgo 'yaṃ akṣaroktas tu nityaśaḥ || 55

[*mātrā-saṃkhyā-vinirdiṣṭo gaṇair mātrāvikalpataḥ |*
"śiṣṭau glāv" iti vijñeyaḥ pṛthag vīkṣya vibhāgataḥ || [56]

mātrāgaṇo guruś caiva laghunī ca vilakṣitahiḥ (?) ǀ
āryāṇāṃ sa caturmātraḥ prastāraiḥ parikalpitaḥ ǁ [57]

prākṛta-prakṛīnāṃ tu pañcamātro gaṇaḥ smṛtaḥ ǀ
vaitālīyaṃ puraskṛtya ṣaṇātrādyās tathaiva ca ǁ [58] [1a]

tryakṣarās tu trikā jñeyā laghu-guru-akṣarānvitaḥ ǀ [1b]
mātrāgaṇa-vibhāgas tu guru-laghv-akṣarāśrayaḥ ǁ] [59] [2a]

II *vṛttāṅga-parimāṇaṃ tu chittvārdhena yathākramam* ǀ
nyasel laghu, tathā saikaṃ chittvārdhena guru nyast ǁ 66 10

III *antyād dviguṇitād rūpād dvir dvir; ekam guror haret* ǀ 2b
dviguṇaṃ ca laghoḥ kṛtvā saṃkhyāṃ piṇḍena nirdiśet ǁ 60 3a

evaṃ vinyasya vṛttānāṃ naṣṭoddiṣṭaṃ vibhāgataḥ ǀ
guru-laghv-akṣarāṇīha sarva-chandassu darśayet ǁ 67 11

IV *ekādhikāṃ tathā saṃkhyāṃ chandaso viniveśayet* ǀ
yāvat pādaṃ tu pūrveṇa pūrayed uttaraṃ gaṇam ǁ 63 6

evaṃ kuryāt tu sarveṣāṃ pareṣāṃ pūrva-pūraṇam ǀ 7a
naidhanād aṅkam ekaikaṃ pratilomaṃ vivarjayet ǁ 64 4b

ādyaṃ sarva-guruṃ jñeyaṃ vṛttaṃ tu sama-saṃjñitam ǀ 3b
kośe tu sarva-laghv-antyaṃ, miśraiḥ śeṣāni sarvaśaḥ ǁ 61 4a

sarveṣāṃ chandasām evaṃ laghv-akṣara-viniścayam ǀ
Va *jānīta sama-vṛttānāṃ saṃkhyāṃ saṃkṣepatas tathā* ǁ 65 5

b *vṛttānāṃ tu samānānāṃ saṃkhyāṃ saṃyojya ṭvatīm* ǀ
rāśya-ūnāṃ ardhaviṣamāṃ samāsād iti nirdiśet ǁ 62 8

c *tad ardhaviṣamānāṃ dviḥ saṃkhyāṃ kṛtvā tu tāvatīm* ǀ
rāśy-ūnām atra jānīta viṣamāṇāṃ samāsataḥ ǁ om. 9
iti chandāṃsi yānīha mayoktāni dvijottamāḥ ǀ
vṛttāny eteṣu nāṭye 'smin prayojyāni nibodhata ǁ 68 XV. 1

To translate these, which are formulated at times in a rather strange manner but are quite intelligible in the light of the foregoing discussion, would mean repeating what has already been said. Hence, I shall give a brief summary of the contents.

After the introductory verse 51, 53-54 describe Hemacandra's first method for the *prastāra*, 55 his second method; 66 deals with *naṣṭa*, 60 with *uddiṣṭa*, 67 summarizes the two and leads us to the next topic. After *naṣṭa* and *uddiṣṭa* now comes *sarvaikādi-ga-la-kriyā*. This term is rephrased in two different ways in our text (67a, 65a). After the description of the method (this corresponds to that of Hemacandra) in 63-64, the result is explained in 61. 65a leads us to *saṃkhyā*, and 65b teaches how to find *sama-saṃkhyā* through addition (of the numbers of the *sarvaikādi-ga-la-kriyā*, cf. Hemacandra's *sūtra* 10). 62 offers the calculation of the *ardha-sama-saṃkhyā*, which must logically be followed by the *viṣama-saṃkhyā*. This, however, occurs only in recension A.

Thus we have before us a system of *Pratyayas* which agrees remarkably with Hemacandra's. In comparison to Piṅgala, there are already some innovations but, as is to be expected of such an old text, there is no mention of moric stanzas. As in Piṅgala, the *ardha-sama* and *viṣama vṛttas* are discussed only in connection with *saṃkhyā*. Finally, it is worth noting that there is total silence about *adhvayoga*.

V. Moric Metres

Now we turn our attention to moric metres, which are considerably more complicated than syllabic metres. In syllabic metres, the two elements of a given form, G and L, have the same value and can be interchanged at will, whereas now in moric metres a G is always equal to 2 L. Above all, this is the fundamental difference between the fixed scheme of syllabic metres and the scheme of moric metres which is variable within certain limits, and this differece must influence the *Pratyaya* theory as well.

When we wish to extend the *Pratyayas* to moric metres, the immediate response would be to arrange a row of *n* morae just as we arranged previously a row of *n* syllables and to write down the same *Pratyayas* for these. But unlike syllabic metres, even a single moric metre like the *Āryā* can have quite many different forms. Therefore, we have here quite a new task that is much more important for the practice, namely to develop *Pratyayas* for individual metres. For this purpose, we should take up the *gaṇa* division of the metre concerned. In the case of syllabic metres, this *gaṇa* division is made quite mechanically for practical reasons; hence it plays just a minor role in the *Pratyayas* as well. On the other hand, in the case of moric metres, the *gaṇas* are the only actual building blocks with which the metre is built up, and here the *gaṇas* are characterised in quite different ways through the varying quantities and a very specific structure. Hence, we should first treat each individual *gaṇa* as a group of *n* morae in the manner first indicated (while observing special rules like the prohibition of ∪—∪), and then construst the *Pratyayas* of the whole metre by combining the *Pratyayas* of all the individual *gaṇas*.

Indian did not go up to this last step. The *Prākṛta-paiṅgala* (henceforth abbreviated as Pp) and Dāmodara stop with the treatment of the set of *n* morae. With great sharpness of intellect, they contructed by brilliant methods, the *Pratyayas* for the group of *n* morae to the same level of perfection as they did for the group of *n* syllables. Of such *Pratyayas*, Hemacandra discusses only one, viz. the *saṃkhyā* for *gaṇas* having any number of morae, in *sūtra* 16. Precisely this very method is wanting in Pp, although *nasta* and *uddista* in this book anticipate it. On the other hand, for each *Pratyaya*, Hemacandra gives in detail the corresponding method for the *Āryā*. But he teaches the *prastāra* for individual *gaṇas* only in the framework of the *prastāra* for the entire *Āryā*. This *prastāra* is derived from the *varṇa-prastāra* in a not very felicitious manner, and is different from the *prastāra* given in the Pp. Thus the difference between the schools of prosody to which Hemacandra and the Pp most have belonged is seen in a particularly conspicuous manner at this place of their system. But the way in which Hemacandra appends the *saṃkhyā* for *gaṇas* of *n* morae at the end also shows that while writing his prosody, he apparently did not hesitate to borrow from the other school as well. From this we get a not unimportant support for what can occasionally be suspected even otherwise while studying his metrical system.

In theory, Hemacandra claims to teach *Pratyayas* for any moric metre; in reality, however, it is only the *Āryā* he deals with, and the verses from his source, fortunately cited by him, are coined expressly for this metre. The only place where we encounter the *Pratyayas* for the *Āryā* and the same verses of the same source once again in Nārāyaṇa's commentary on the *Vṛttaratnākara*, as we have mentioned above. That Nārāyana, though much later than Hemacandra, is not dependent on him follows from the fact that his treatment lacks Hemacandra's *sarvaikādi-ga-la-kriyā;* had he copied from Hemacandra, he would not have dropped it by any means. On the other hand, since it is precisely to this method that Hemacandra does not cite any verses from the source, it follows that the common source of Hemacandra and Nārāyaṇa did not know this method. Hence, Hemacandra, who certainly did not invent it himself, must have taken it from some other source. Probably he borrowed everything from a younger elaboration of Nārāyaṇa's source.

For the time being we cannot say anything more about this source; its knowledge would be of greatest interest in understanding Hemancandra's prosody. However, that this system of *Āryā Pratyayas* represents an isolated bylane and is in no way connected to the moric *Pratyayas* of the Pp, nor to one of them discussed by Hemacandra himself, is indicated more clearly by Nārāyaṇa through the place which he assigns to the *Āryā Pratyayas*. Instead of including them in the section on the *Pratyayas* where they indubitably belong — this section occurs in Kedāra's book also at the end and forms the chapter VI — he inserts them in his treatment of the *Āryā* metre in the chapter II.

We need not make here any further remarks about Hemacandra's *Āryā-prastāra* and the *saṃkhyā* calculation developed in this connection because both are intelligible and become clear without much ado.

The *naṣṭa* method is a clever adaptation of the *varṇa-naṣṭa* to the changed circumstances; this will be evident through a few brief remarks. If we designate the five possible forms of the *gaṇa* (of which the third does not occur in old *gaṇas*) with a, b, c, d, e, then the *prastāra* for 4 gaṇas will commence in the following manner.

a a a a	a d a a	a b b a
b a a a	b d a a	b b b a
d a a a	d d a a	d b b a
e a a a	e d a a	e b b a
a b a a	a e a a	and so on, all in all,
b b a a	b e a a	400 forms for 4 *gaṇas*.
d b a a	d e a a	
e b a a	e e a a	
a c a a	a a b a	
b c a a	b a b a	
d c a a	d a b a	
e c a a	e a b a	

As we can see, this scheme can be divided, first of all, into groups of four lines each, in which the first *gaṇa* has the forms a,b,d,e, in a regular order of succession. For determining the form of the first *gaṇa,* therefore, we have only to find out which position within the group is occupied by the desired line. This is done by calculating (i.e. by dividing the given number by 4) how many complete groups of 4 precede our line and how many individual lines still remain. Here the numbers that are divisible by 4 without leaving a remainder indicate, as one glance at the table shows, directly the fourth position, i.e. form e. After thus determining the first *gaṇa,* three lines can now be removed from each group, and we obtain a new table (as in Fig. 9 above) consisting of groups of 5 lines each. In order to find out which serial number in this new table is borne by our line, we must determine in which group of 4 in the old table it stood. Since the result of dividing by 4 gave us the number of preceding groups, the quotient must now be augmented by 1 so that we can take into account that group also to which the remainder after the division belongs (and thus the required line as well). This is naturally unnecessary, when the division leaves no remainder. The new number thus found must be divided by 5, corresponding to the groups of 5 lines in the new table, in order to determine the form of the second *gaṇa,* and so on.

After these hints, we can leave it to the reader to work out by himself the reverse process in the *uddiṣṭa.*

Finally, the *sarvaikādi-ga-la-kriyā* is far simpler than it appears. Let us designate the three "types" of the *gaṇa* (2G; 1G2L; 4L) as a,b,c. Then for the first two *gaṇas* we have the possibilities: 1a, 2b, 1c; 1a, 3b, 1c. While in the case of the *saṃkhyā,* in order to combine each possibility of the first *gaṇa* with each of the second, we multiplied summarily the 4 of the first with the 5 of the second, now in order to obtain the same result, we should multiply each individual group of the first *gaṇa* with each individual group of the second *gaṇa*. This will result in

1 a.a	2 a.b	1 a.c
3 a.b	6 b.b	3 b.c
1 a.c	2 b.c	1 c.c

What do these numbers mean?

a.a = 2G. 2G	= 0L, 4G
a.b = 2G. 1G, 2L	= 2L, 3G
a.c =	= 4L, 2G
b.c. = 1G, 2L. 4L	= 6L, 1G
b.b = 1G, 2L. 1G, 2L	= 4L, 2G
c.c = 4L .4L	= 8L, 0G

Thus it turns out that b.b. is the same as a.c. Hence in the original table we can write the two sets of a.b. and the two sets of b.c. together and also combine the two sets of a.c. with b.b. At the same time, it becomes clear how this process leads by itself to the *kapāṭa-sandhi* principle, according to which the necessary additions can be performed at once mechanically. In our case, the result is

1 × 0L, 4G
5 × 2L, 3G
8 × 4L, 2G
5 × 6L, 1G
1 × 8L, 0G.

With this the procedure of calculations must have become clear, and there is no need to dwell upon it further. Although this as well as the other *Āryā Pratyayas* cannot claim for themselves such universal mathematical interest as some other *Pratyayas* discussed earlier in connection with syllabic metres do, yet we cannot deny recognition here also to the ingeniousness and elegance with which different problems are solved.

VI. Prākṛit Prosody

The Pp presents the *Pratyayas* in the following arrangement, the selection and the sequence of which is rather peculiar.

mātrāṇām uddiṣṭam
„ „ *naṣṭam*
varṇānām uddiṣṭam
„ „ *naṣṭam*
varṇameruḥ, varṇapatākā
mātrāmeruḥ, mātrāpatākā
vṛttasya laghu-guru-jñānam[39]
sakala-prastāra-saṃkhyā.[39]

The most conspicuous in this list is the absence of any kind of *prastāra,* which forms the starting point in other texts. Of the other processes, the *varṇoddiṣṭa* corresponds to the second method which is preferred by Kedāra; *varṇa-naṣṭa* and *varṇa-meru* are as in Piṅgala; *patākā* and *laghu-guru-jñānam* will be discussed briefly at the end (the latter, it may be mentioned right here, has nothing to do with *sarvaidādi-ga-la-kriyā*). Finally, *sakala-prastāra-saṃkhyā* has the limited task of calculating the total number of forms of all metres containing 1 to 26 syllables, i.e. 134217726.

As already stated, the three actual moric *Pratyayas* are not accompanied by the *prastāra* which, however, they anticipate and without which *naṣṭa* and uddiṣṭa would remain unintelligible even here. Fortunately Nārāyaṇa fills this lacuna. His method is the logial adaptation, to the changed circumstances, of the second method of *varṇa-prastāra* which Hemacandra discusses as the first. We begin with all G, and write in all the subsequent forms a L always under the first G; all that follows remains unchanged. We go on filling the missing morae, proceeding towards the left, by G. If the line is short by an odd number of morae, we insert appropriate number of Gs plus one L to their left. Thus, with the same considerations as the *varṇa-prastāra,* this table is continued systematically and completed. Thus for 6 morae we get the table given in Fig. 11.

— — —
U U — —
U — U —
— U U —
U U U U —
U — — U
— U — U
U U U — U
— — U U
U U — U U
U — U U U
— U U U U
U U U U U U

Fig. 11

Both *naṣṭa* and *uddiṣṭa* are based on a peculiar series of numbers, where each term is the sum of the two preceding terms (1, 2, 3, 5, 8, 13, 21 ...). We became familiar with this series in connection with Hemacandra's *saṃkhyā* for *n* morae. Henceforth we call it moric series.

a) *Naṣṭa:* To find out the form of the *n*-th line of the *prastāra*, write down all the morae singly in a row and above them the terms of the moric series. Then subtract the given number from the number above the last mora; from the difference subtract the number preceding the last. If this number is too large for subtraction, skip it and go to the one previous to it. Thus go through all the numbers. Where subtraction is possible, the mora under that number forms, in conjunction with the next mora, a G; the remaining morae are L.

Example: What is the form of the 7th line in the *prastāra* table for 6 morae?

1 − 1 = 0			6 − 5 = 1		13 − 7 = 6
1	2	3	5	8	13
u	u	u	u	u	u
—		u	—		u

Result: form no. 7 is — u — u.

b) *Uddiṣṭa:* Given an arbitrary form, what is its serial number in the *prastāra* table? Write down the given form and also the terms of the moric series in the following manner. For each G (consisting of 2 morae) write down one term above it and one below it, and for each L only one term above it. Then add the numbers above all the Gs and subtract their sum from the last number. The difference is the serial number required.

Example: What is the serial number of the form — u u — in the *prastāra* table for 6 morae?

1	3	5	8
—	u	u	—
2			13

1 + 8 = 9; 13 − 9 = 4; hence, form — u u — is no. 4.

It is obvious that here also both the processes find their justification in the peculiar construction of the *prastāra* table. In actual fact, this table is constructed on the basis of the moric series. This will become more clear, if we rewrite it from right to left, arranging it according to syllables as though it were a *varṇa-prastāra* (see Fig. 12). Then we notice the following:

Throughout the table, there is always a smaller group of G and a larger group of L vertically one above the other, and these two groups correspond to two successive terms of the moric series; to the left of each of these groups, there are two smaller groups, again G above and L below, their numbers corresponding to two successive terms of the moric series, their sum being the larger group to their right. Thus the last column of the table consists of 5G + 8L; next to 5G are 2G + 3L; next to 8L we have 3G + 5L; proceeding towards left, wherever we find 3G or 3L one above the other, to their left are always 1G + 2L. To the left of each group of 2G or 2L, we find always 1G + 1L. Hence, preceeding from the right towards left, from the largest group to ever smaller groups, we can find, through the given number, the location of one syllable after the other within a specific group and thus its form.

— — —
∪ ∪ — —
∪ — ∪ —
— ∪ ∪ —
∪ ∪ ∪ ∪ —
∪ — — ∪
— ∪ — ∪
∪ ∪ ∪ — ∪
— — ∪ ∪
∪ ∪ — ∪ ∪
∪ — ∪ ∪ ∪
— ∪ ∪ ∪ ∪
∪ ∪ ∪ ∪ ∪ ∪

Fig. 12

In our example, the form of the 7th line is wanted. A glance at the table shows that its last syllable lies within the group 8L. But, since we should always proceed systematically from the largest group to ever smaller groups, it is not practical to determine the nature of the last syllable directly by saying: the 7th line lies beyond the first 5G (this would be a step forward whereas the whole process is directed to a backward movement). Instead of this, by subtracting the given number from the total number (i.e. the number written above the last mora), we determine that there are still (13 − 7 =) 6 lines to follow our 7th line. The number 6 is smaller than 8 (i.e. 8 cannot be subtracted from 6); hence our line lies within the last 8L, which fact indicates that the last syallale is a L. To the left of 8L, there must be first 3G and below it 5L. Since 6 more follow it, our line is the 7th from the bottom, hence it does not lie within the 5L but in 3G (which fact denotes that the penultimate syllable is a G). Again within the group of 3G, our line is the second below because (6 − 5 = 1) 1 line follows it. To be left of the 3G, there must be 1G, 2L; hence our line is assigned to the last 2L (hence the third syallable from the last is L). To the left of the 2L, there are 1G and 1L. Since our line is followed by one more (last time we remained in the same group, "performed no subtraction"), it results in the first of the possibilities, that is to say, G. Thus the scansion is found to be — ∪ — ∪. This is the essence of the process, which the Indians made somewhat more mechanical and thus deliberately obscure.

After what has been said just now, a brief hint will suffice for understanding the *uddiṣṭa*. It is based on the principle that, proceeding upwards, from the bottom, group after group is excluded (i.e. subtracted from the total number of lines) until the number sought remains as the rest. The total number of lines is the last term of the moric series, be it the number written below a terminal G or above the terminal L. Now it will be clear that one or more terminal Ls leave the possibilities open until the last (just as all Ls form the very last possibility), whereas a terminal G at once excludes the last 8 possibilities. G at the penultimate position with a terminal L excludes the last 5 possibilities, with a terminal G, excludes 8 + 3 = 11 lines, and so on. Hence, subtraction takes place only with the long syllables. The Indian way of distributing the terms of the moric series on the scansion of a given line implies that the number written above G always represents the group to be excluded.

Now finally, corresponding to the *meru-prastāra* of the syllabic metres, a number triangle is constructed for groups of *n* morae also, from which the number of the *sarvaikādi-ga-la-kriyā* can be read out. Here too the construction goes on quite mechanically, but it is slightly more complicated than in the case of the syllabic metres.

Draw a square, below it 2 horizontal rows consisting of 2 squares each, then 2 rows of 3 squares each, and so on, increasing 1 square for each 2 rows, and that too in such a way that all the last squares of the row lie one below the other. Then these squares are filled as follows: Write 1 in the topmost square and in all the last squares.

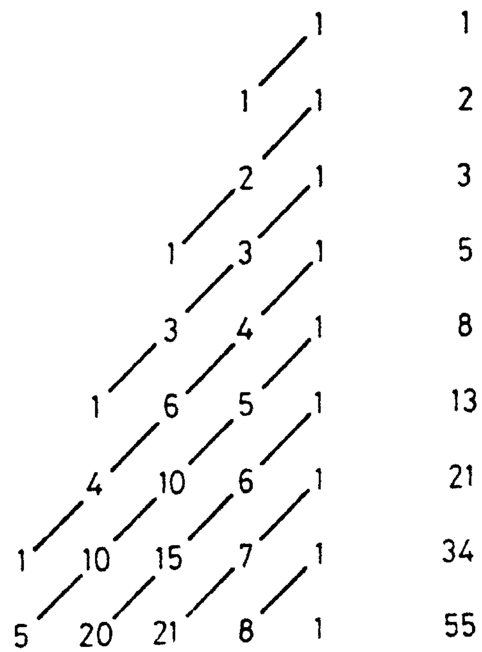

Fig. 13

In the first square of each row come alternately 1 and the natural numbers, i.e. 1,2; 1,3; 1,4; 1,5; .... But in each of the middle squares comes the sum of the number just above it and the number diagonally to the right above the second number. Thus the triangle in Fig. 13 is constructed, which Nārāyana calls *mātrā-khṇḍa-meru* in contradistinction to the simple *mātrā-meru*. He constructs the latter with the same numbers but as an isosceles triangle, and the rule for filling up the middle squares here is still slightly more comlicated. Now in our triangle, the 6th line, for example, shows that the following can be constructed with 6 morae: 1 *pāda* with 3G; 6 *pādas* with 2L, 2G; 5 with 4L, 1G; and 1 with 6L. If we add numbers of each horizontal row, we get the moric series as shown in Fig. 13.

The mathematical principles underlying this process are no doubt more involved and difficult than in the case of other processes but, on the other hand, being special cases and derivatives, are not so interesting mathematically as the syllabic *Pratyayas*. For this reason and for reason of space, I must limit myself to a few remarks just to show in which direction the solution must be sought for the problems that arise.

If we connect the numerals in Fig. 13 in the manner indicated there, we will easily see that we have in front of us a distorted variety of Pascal's triangle. This "distortion" has the following reasons. In the syllabic *sarvaikādi-ga-la-kriyā,* all groups for a specific number of syllables consist of just so many elements, i.e. precisely the number of syllables of that group. Not so for *n* morae. If we begin with Ls only,[40] then in each subsequent group there will be 2L less but, on the other hand, 1G more, thus together 1 element less than in the preceding group. For example, with 6 morae, we have

group 1	6 L	= 6 elements		
„ 2	4L, 1G	= 5	„	
„ 3	2L, 2G	= 4	„	
„ 4	3G	= 3	„	and so on.

Therefore, while in the case of the syllabic metres, all groups for a specific number of syllables belong to one and the same binomial series, in the case of moric metres, each successive group of the *sarvaikādi-ga-la-kriyā* is a term of the binomial series having an exponent less by 1 each time. To take a practical example, in Fig. 13, line 7, the first 1 (reading backwards) belongs to the binomial series of the 7th order of Pascal's triangle, the next 6 to the binomial series of the 6th order, the next 10 to the 5th, the next 4 to the 4th, and so on.

Now on the basis of the *mātrā-meru,* we can also show that each term in the moric series equals the sum of the two preceding terms and we can also show the reason why it is to. This will be done with the help of the same formula with which we have proved earlier that in Pascal's triangle each number is equal to the two numbers standing above it. When we rewrite once again the individual numbers of the triangle in the form of divided factorials and so arrange the figure that the terms with the same base are always one below the other, we get the following figure.

$$\begin{array}{cccccccc}
 & & & & & & & 1 \\
 & & & & & & 1 & 1_1 \\
 & & & & & 1 & 2_1 & \\
 & & & & 1 & 3_1 & 2_2 & \\
 & & & 1 & 4_1 & 3_2 & & \\
 & & 1 & 5_1 & 4_2 & 3_3 & & \\
 & 1 & 6_1 & 5_2 & 4_3 & & & \\
1 & 7_1 & 6_2 & 5_3 & 4_4 & & &
\end{array}$$

Now all the divided factorials in which the base and index are the same (e.g. 4_4) are equal to 1 and hence can be interchanged at will. Keeping this in mind, if we add, e.g. the lines 6 and 7 according to the formula

$$n_p + n_{p+1} = (n+p)_{p+1}$$

We get

$$\begin{array}{rcccl}
1 & & & & 1 \\
6_1 + 1 & = & (6+1)_1 & = & 7_1 \\
5_2 + 5_1 & = & & & 6_2 \\
4_3 + 4_2 & = & & & 5_3 \\
3_3 & = & 1 & = & 4_4
\end{array}$$

That is to say, the series resulting from the addition of the lines 6 and 7 constitute the 8th line of the *mātra-meru*. Of course, by substitting n for the numbers, we can easily produce the general proof that the rows n and $n+1$ added together yield the row $n+2$. In other words, in the moric series whose terms are the sums of individual rows of the *mātrā-meru,* two terms added together always produce the next term. This is the mathematical rationale of the *saṃkhyā* rule for *gaṇas* of n morae, a rule given only by Hemacandra.

VII. Minor Pratyayas

Finally, we will discuss two more *Pratyayas, patākā* and *markaṭī,* just for the sake of completeness. Of these, the *patākā* is still described in the text of the Pp but the *markaṭī* only in some verses interpolated between I. 49 and 50, and this very fact shows that it is the latest of all *Pratyayas.* Attempts to complete the system — to which basically nothing significance could be added — in all conceivable fullness led to the origin of these offshoots which cannot be described either as very interesting or particularly ingenious.

Markaṭī is a table which displays, in 6 horizontal lines, the number of syllables or morae, as the case may be, of the metre concerned, the number of combinations that can be formed out of them (i.e. the *samkhyā*), the number of the morae contained in the entire *prastāra* table, the numbers likewise of syllables, Ls and Gs. This table is elevated to the status of *Pratyaya* only because of the manner in which one row is mechanically derived from the other through multiplications, additions etc., the logic of which is often easy to understand. The *laghu-guru-jñāna* of the Pp contains a kernel of this process. It is rather a primitive speculation about the numerical proportions between the morae, syllables, Gs and Ls in a metre. The number of Gs is found by subtracting the number of syllables from that of morae (since for each G, there must be 1 mora more than the syllables); the number of Ls is obtained by subtracting the number of G from that of syllables. These relations are built into a whole system in Ratnākaraśānti's *Chandoratnākara* (ed. Huth, p. 19, vv. 17, 20-23). All the statements there refer to the entire table of *varṇa-prastāra*. In the table, the number of syllables (S) = the number of syllables in a metre multiplied by its *saṃkhyā*. The number of G and L = 1/2 each of this number. The number of morae (M) = one and half times S. For example (the text has none), in a metre of 4 syllables, S = 4 × 16 = 64; G and L = 32 each; M = 64 + 32 = 96. Now in verses 20-23, all possible equations between these four quantities are given.

G: 1) $S - L = 64 - 32 = 32.$
2) $\frac{M - L}{2} = \frac{96 - 32}{2} = 32.$

L: 1) $M - 2 \times G = 96 - 2 \times 32 = 32.$
2) $2 \times S - M = 2 \times 64 - 96 = 32.$
3) $S - G = 64 - 32 = 32.$

S: 1) $G + L = 32 + 32 = 64.$
2) $M - G = 96 - 32 = 64.$
3) $\frac{M + L}{2} = \frac{96 + 32}{2} = 64.$

M: 1) $L + 2 \times G = 32 + 2 \times 32 = 96.$
2) $2 \times S - L = 2 \times 64 - 32 = 96.$
3) $S + G = 64 + 32 = 64.$

However, *patākā* is somewhat more interesting than these calculations. It is dealt with as a sort of appendix to *sarvaikādi-ga-la-kriyā*. Subsequent to the formation of groups for individual forms in *sarvaikādī-ga-la-kriyā*, the question is raised as to how they distribute themselves in the *prastāra* table, i.e. which numbers in the *prastāra* table have the forms with a specific number of G and L. Nārāyaṇa teaches the method for syllabic metres only, the Pp does so for moric metres as well. All the numbers of the *prastāra*, arranged according to the groups of the *sarvaikādi-ga-la-kriyā*, are built into an acute isosceles triangle with the base upwards. This figure explains the name *patākā*, "pennent" or "banner", while there does not seem to be available any such explanation for the strange name *markaṭī*. As a specimen, I will just give below the patākā for 5 syllables according to Nārāyaṇa.

First write down the series 1,2,4,8,16 ... up to one more term than there are syllables (thus in our example, 6 terms). Then add the first 1 successively to all the following terms of the series and write down the sums in a vertical column under the 2 occupying the second position. Repeat the same with 2, i.e. and the 2 to the following terms of the series and write down the results under 4. After that, add likewise one after the other all the numbers below the 2 to all the terms following 2 in the series and write these sums below the numbers already written under 4. Only then, add the 4, and after that the numbers under the 4, to the terms following 4 in the series, and so on. In all these additions, two basic rules must of course be observed: if a number is obtained which is larger than the last of the first horizontal line (which after all indicates the highest available number) this is dropped, and a number already available is not written for the second time. Because of these rules, we get the table as shown in Fig. 14. In this table, the vertical columns correspond to the individual groups of the *sarvaikādi-ga-la-kriyā*. For instance, the fourth column indicates that the forms with 2G, 3L occur at the 8th, 12th, 14th, 20th, 22nd, 26th, 15th, 23rd, 27th and 29th places of the *prastāra* table.

1	2	4	8	16	32
	3	6	12	24	
	5	10	20	28	
	9	18	14	30	
	17	7	22	31	
		11	26		
		19	15		
		13	23		
		21	27		
		25	29		

Fig. 14

We have now reached the conclusion. As stated already at the beginning, the theory of *Pratyayas* never gave up its connection to prosody, or rather this association became more pornounced in later times. Therefore, the mathematical hopes aroused by these beginnings were not quite fulfilled by the later developments which got lost in special cases. But, apart from its creative influence on mathematics which we could prove in two cases — there may be more such but we did not follow these further at present — and apart from the clarifications or confirmations it offers on the question of individual writers on prosody or schools of prosody, the theory of Pratyayas contains without doubt a very remarkable amount of mathematical perceptions; but as a whole it is, in any case, a peculiar as well as typical product of the Indian mind.

NOTES AND REFERENCES

[1]Henceforth we use the abbreviations L = *laghu* = short syllable; G = *guru* = long syllable.
[2]*Niedere Analyse* by H. Schubert (Sammlung Schubert), pt. 1: Combinations etc., p. 4 ff.

[3]"Über die Metrik der Inder: zwei Abhandlungen" von A. Weber, *Indische Studien: Beiträge für die Kunde des indischen Alterturms*, hrsg. Albrecht Weber, VIII Band, Berlin 1863; reprint: Hildesheim/New York 1973. Pp. 424-457 contain **10** Darstellung und Berechnung der möglichen Combinationszahl eines Metrums. — Tr.

[4]*Prakāśitaṃ Devakaraṇena Śreṣṭhinā Mūlacandrātmajena,* Bombay, Nirṇayasāgar Press, 1912.

[5]"La Métrique de Bharata," *Annales du Musée Guimet,* T. II.

[6]Printed in the edition of the *Vṛttaratnākara* by Vaidyanāth Śāstrī Varakala, Kashi Sanskrit Series 55, Benares 1927.

[7]The tables that will follow — as long as there is no other remark — are added by me for the sake of better overview.

[8]Nārāyaṇa (on *Vṛttaratnākara* II.2) reads: 1a *prathame,* 1b *jñeyo.*

[9]This should probably be emended as *gy ekaṃ tyajet.*

[10]On this reasoning, cf. p. below.

[11]Nārāyaṇa, on *Vṛttaratnākara* II. 2, reads: *ghāta.*

[12]Thus reads Nārāyaṇa correctly; Hemacandra has *haṭādvāryo°.*

[13]On the analogy of *trimātra* and *caturmātra* that will follow in the commentary, we should understand *mātrāṇam* as a *Bahuvṛīhi* compound without the first term. The first term ought to be an arbitrary number (we would say today n) for which no special word was yet available. The present manner of expressing this notion is not uninteresting.

[14]Gaṅādāsa dismisses them in yet stronger words, c.f. p. , n. 30 below.

[15]Halāyudha: *so 'tyalpatvāt puruṣecchānuvidhāyitvenāniyatatvāc ca noktaḥ.* Weber's translation (p. 434), "because of the various differences among the people, it is too uncertain" is not quite right. What is meant is simply this: some people write big letters, others small letters; some write wider letters and others narrower letters. Hence, even after calculating the number of the lines, the space actually needed remains uncertain.

[16]Or rather "*glo*". However, the change of *ā* to *o* is unnecessary in view of what we shall state presently. It is then on obvious case of orthographical error (*glā* for *gglā*).

[17]Yet, on the other hand, Piṅgala teaches a method for calculating the *saṃkhyā* which makes sense only when the number of syllables is much larger than 3.

[18]Thus a modern text book, though it follows otherwise the same principle as the Indians when computing the variations of n elements, interchanges here the first two columns. The table obtained thus is decidely inferior to the Indian table.

[19]"The word *sajātīya* can mean only the short syllables? cf. the employment of *vijātīya* in the commentary on 22, p. 428 above." — Weber's footnote.

[20]"This puzzling sentence occurs exactly in the same wording again below in the commentary on rule 30 (p. 446). Probably it contains a philosophical justification for commencing the calculation with the number 1?" — Weber's footnote.

[21]E.g. what is the serial number of the form — ∪ ∪ — ∪ in the *prastāra* of 5 syllables? Answer 1 2 4 8 16 (— ∪ ∪ — ∪); $2+4+16=22$; the serial number sought is $22+1=23$.

[22]In modern mathematics, this is stated in general terms thus: the variation of n things of p-th class with repetition yields n^p complexes, $\overset{w}{V}_{p}(n) = n^p$. In this formula, we should substitute $n = 2$ for our special case; then it will be identical to Hemacandra's rule.

[23]The questions posed here have been dealt with in the preceding discussion. Piṅgala's *sūtras* (28: *dvir ardhe;* 29: *rūpe śūnyam;* 30: *dviḥ śūnye;* 31: *tāvad ardhe tad guṇitam*) are translated by Weber thus: "Twice at the half. At 1 a zero. Twice at a zero. At the half, multiplication of the same once as much (square of it)." I think the following rendering brings out the actual procedure more faithfully: "While halving, (note down) a 2; while (subtracting) 1 (note down) a zero. For (each) zero, doubling (takes place); where (the number) was halved, squaring (takes place)."

[24]It may be recalled that, according to the law of exponents, $a^x = a^{x-1} . a$ and $a^x = (a^{x/2})^2$. That is to say, if we subtract 1 from the exponent, to make up for it we must multiply with the base (in our case, with 2) (this is indicated by the index number 0); if the exponent is halved, then in order to make up for that we must square the whole thing (index number 2).

[25]From Colebrokke's *Algebra*, Weber (449 f.) cites a passage from Pṛthūdaka's comentary on Brahmagupta's *Brāhmasiddhānta*, whence it become clear that Piṅgala's method was adopted by the mathematicians. They used it for calculating the sum of the geomatric progression. In this connection, Pṛthūdaka expressly refers to the origin of this method in prosody.

[26]Owing to the constraints on space, I must refer the reader to text books on elementary analysis for example and for graphic representation.

[27]M. Cantor, *Vorlesungen über die Geschichte der Mathematik*, vol. 2, p. 751 ff.

[28]Of course, the writers on prosody did not advance this far, but here also the mathematicians apparently took up the problems posed by prosody and developed them further. Thus in Bhāskara's *Līlāvatī, we find a method for sarvaikādi-ga-la-kriyā,* with an express mention that it is meant for prosody. This method actually goes beyond the formation and calculation of divided factorials. Nārāyaṇa took up the same method and commented on it. For details, see Weber, pp. 456-7.

[29]See also Michael Hahn (ed.), *Ratnākaraśānti's Chandoratnākara*, Kathmandu 1982 (Nepal Research Centre, Miscellaneous Papers, No. 34), esp. Introduction, pp. 3-6. — Tr.

[30]Yet it is missing in the *Śrutabodha*, Kṣemenra's *Suvṛttatilaka*, and Gaṅgādāsa's *Chandomañjarī*. The last one dismisses this topic with the remark (Vi.5) *prastārādi punar noktam, kevalaṃ kautukam hi tat,* "but we have not discussed *prastāra* and the like, for they are nothing but curiosities".

[31]The French edition by Grosset (chs. I-XIV), Paris 1898, and a new edition with commentary in GOS (36), Baroda 1926, vol. 1, Adhyāya 1-8.

[32]"La metrique de Bharata, "*Annales du Musée Guimet*, t. *II*.

[33]*Thus, e.g. he translates as Piṅgala's prastāra* what in reality is a different method taught by Hemacandra.

[34]Kashi Sanskrit Series, No. 60, Benaras, 1929.

[35]Kāvyamālā 42, ed. Sivadatta and Parab, Bombay 1894.

[36]However, the ms. *kha* used for *Be* belongs to recension A.

[37]These verses refer to *mātrā-gaṇas* and even to *mātrā-prastāra*. But of moric metres, Bharata discusses rather briefly just the *mātrāsamaka* and *Āryā* at the end; in the introductory section (R 1-50) he does not refer to them at all (R 50b) is wrongly positioned by Regnaud; it can only refer to *Daṇḍaka*). Hence, when *Vaitālīya* occurs in R 58, which is not discussed by Bharata at all, and when mention is made of *gaṇas* also with 5 and 6 morae, it is quite certain that these verses, partly corrupt beyond measure, must be late interpolations like 52 which speaks of *mātrāprastāra*.

[38]Variant readings from three editions of the *Nāṭyaśāstra*, given in the original, are not reproduced here, as more editions have come out since the article appeared in 1933. Notable are *Nāṭyaśāstra with the commentary of Abhinavagupta*, ed. M. Ramakrishna Kavi, vol. II, Baroda 1934 and *The Nāṭyaśāstra, ascribed to Bharata-Muni*, ed. Manomohan Ghosh, vol. I, Calcutta 1967. Baroda edition XIV. 116-133 deals with the *Pratyayas;* the text and Abhinavagupta's commentary deserve close study. Calcutta edition omits this section. However, Baroda XIV. 55-81 and Calcutta XV. 52-79 enumerate the *samkhyā of the samvṛttas,* having 1 to 26 syllables and their sum. — Tr.

[39]These two *Pratyayas* are not mentioned in Dāmodara's *Vāṇībhūṣana* which, incidentally, does not enjoy any independet status after Piṅgala's work.

[40]Contrary to this practice, Indians begin with just G. Therefore, Fig. 13 has to be read even afterwards from right to left.