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The Pāvulūrigaṇitam
The first Telugu Work on Mathematics

I

Although specimens of Telugu prose and verse can be seen in inscriptions dating from the 6th century A.D. on,¹ Telugu came into its own as a medium of literary composition in the 11th century under the active patronage of the Eastern Cālukyas of Veṅgī. The first extant Telugu work is the translation of the Mahābhārata (Ādi, Sabhā and a part of Aranya parvan) by Nannaya Bhaṭṭa, the court poet of Rājarāja (1022-1063) of the Eastern Cālukya dynasty.

The inspiration for composition in Telugu instead of Sanskrit came apparently from Karṇāṭaka where writing in Kannaḍa began more than a century earlier, for there was an active exchange of scholars and poets between these two regions. The first two great poets in Kannaḍa, Pampa and Ponna, hailed from Veṅgī. Pampa migrated to the court of Arikesarin (mid 10th c.) at Vemulavāḍa and wrote there the Vikramārjunavijaya and the Ādipurāṇa in Kannaḍa.² His Kannaḍa rendering of the Mahābhārata is said to have inspired Nannaya to make a similar effort in Telugu. Ponna, on his part, received the patronage of the Rāṣṭrakūṭa ruler Kṛṣṇa III (10th c.) and composed the Śāntipurāṇa in Kannaḍa.³ On the other hand, Nārāyaṇa Bhaṭṭa, whose assistance in the composition of the Telugu Mahābhārata is gratefully acknowledged by Nannaya, is said to be an envoy of the Western Cālukya monarch Someśvara I (1042-68) to the court of Rājarāja.⁴

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1. Divākarla Veṅkaṭāvadhāni, Andhra-vāṇmayārambhadaśa, vol. I: Prāṇ-Nannaya-yugamu, Hyderabad 1960, p. 88ff.
 2. Ibid., p. 93. Pampa is said to have written also a Telugu work called Jinendrapurāṇamu which is no more extant.
 3. Ibid., pp. 93-94, 102. A Telugu work entitled Ādipurāṇanam is attributed to Ponna as well, but this is not extant either.
 4. B. S. L. Hanumantha Rao, "Sources of Andhra History" in: S. P. Sen, ed., Sources of the History of India, vol. I, Calcutta 1978, p. 124.

This urge to translate Sanskrit works into regional languages and the vigorous exchange of scholars between Telugu and Kannaḍa regions is reflected also in the translation of scientific texts, especially those on mathematics, into Telugu and Kannaḍa. To cite some prominent cases: Mahāvīra's Gaṇitasārasaṅgraha (GSS),⁵ written in Karṇāṭaka during the rule of the Rāṣṭrakūṭa king Amoghavarṣa Nṛpatuṅga (814-880), was translated into Telugu by Pāvulūri Mallana at the end of the 11th century. About a century later, under the reign of the Hoyasala king Ballāla II (1173-1220), Rājāditya rendered into Kannaḍa the Līlāvati,⁶ soon after its composition by Bhāskara II some time in the middle of the 12th century. Daivajña Vallabha wrote commentaries on the GSS both in Kannaḍa and Telugu,⁷ and a Vallabhācārya translated the Līlāvati into Telugu in the 16th century while Acyutarāya ruled at Vijayanagara.⁸

Of these works, the Telugu rendering of the GSS by Pāvulūri Mallana, known popularly as the Pāvulūriṅgaṇitamam (PG), is perhaps the earliest translation of a scientific work from Sanskrit into a popular speech and also the second extant work in Telugu, but it has not received the attention it preeminently deserves.⁹

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5. The Gaṇita-sāra-saṅgraha of Mahāvīrācārya, with English Translation and Notes, by M. Raṅgācārya, Madras 1912; Mahāvīrācārya's Gaṇitasāra-saṅgraha (An Ancient Treatise on Mathematics), authentically edited with a Hindi Translation and Introduction etc. by L. C. Jain, Sholapur 1963. I refer to the first edition as GSS and the second GSS-J.
 6. David Pingree, Jyotiḥśāstra: Astral and Mathematical Literature, Wiesbaden 1981, p. 62.
 7. Thus Vallabha himself, according to Raṅgācārya in his preface to the GSS, pp. viii-ix. Mss. with the Kannaḍa commentary are available, but I have not come across any with the Telugu commentary. For the former, see David Pingree, Census of the Exact Sciences in Sanskrit, A-4, Philadelphia 1981, s.v. Mahāvīra.
 8. Cf. A Descriptive Catalogue of Telugu Manuscripts in the Government Oriental Manuscripts Library, Madras, vol. X: Gaṇita and Jyotiṣa, Madras 1949, ms. no. 2280, p. 2569. It is not known if this Vallabhācārya is identical with the Daivajña Vallabha mentioned above.
 9. As early as 1814, Benjamin Heyne translated the Kṣetragaṇita chapter of the PG into English, on the basis of a ms. the contents of which have little in common with the corresponding chapter of the GSS; see his Tracts, Historical and Statistical, on India, with Journals of several Tours through various Parts of the Peninsula, London 1814, pp. 172-180: "A Free Translation of the Chetri Gaṇitam, or Field Measuring of the Hindoos." Dr R. B. Barnett, Charlottesville, has kindly sent me a xerocopy of these pages from London.

II

Mallana furnishes some information about himself at the beginning of the PG. He was a follower of the Apastamba-sūtra and belonged to the Gārgya-gotra. His parents were Śivvana and Gaurama. His grandfather Malla(na) received the village Navakhaṇḍavāda near Pīṭhapura as an *agrahāra* from Rājarāja.¹⁰ Thus the grandson Mallana was, in all probability, a younger contemporary of Nannaya and may have written the PG some fifty years after the translation of the Mahābhārata, i.e. at the end of the 11th century.

Mallana states that he is the *vibhu* of Pāvulūru in Kammanāḍu. According to Telugu historians, the term *vibhu* here does not mean "lord of the village" but refers to the hereditary post of *karaṇamu*, the keeper of the land records of the village. Pāvulūru is identified with the modern village of the same name in Bapatla taluk of Guntur district in Andhra Pradesh.¹¹ Mallana may have received the surname Pāvulūri (lit. "of Pāvulūru") because he was the *karaṇamu* of Pāvulūru.¹² The keeper of the land records has to measure the individual land holdings, prepare the revenue accounts and be an expert in all sorts of calculations. It is perhaps this professional interest coupled with the popularity of the GSS which may have prompted him to translate the GSS into Telugu.

III

Mallana seems to have named his translation Sarasaṅgrahaṇita after the Sanskrit original.¹³ But it is known more popularly as the Pāvulūrigaṇitam

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10. An inscription of 1181 (Epigraphia Indica, iv, pp. 32-54) records the grant of the same Navakhaṇḍavaḍa to the temple of Kuntimādhava at Pīṭhapura by Jayāmbikā, which fact suggests that the descendants of the elder Mallana may have given up the *agrahāra*. Cf. Cāgaṇṭi Śeṣayya, Āndhra-kavi-taraṅgiṇi, vol. I, second edn., Kapileśvarapuramu 1955, p. 204.
11. *Ibid.*, pp. 201-202.
12. Until its recent abolition, the post of *karaṇamu* was held by the brahmins of the Niyogi subcaste and their surnames are often the same as the names of their villages.
13. Cf. GSS I.19:
kiñcid uddhṛtya tat sārāṇ vakṣye 'haṃ matīśaktitaḥ /
alpāṇ grantham analpārtham gaṇitam sārasaṅgraham //
 Mallana too speaks of Sārasaṅgraha-ḡaṇita in the very second stanza and the colophons in several mss. use this name.

after the surname of the author. Some mss. refer to it also under a third name Daśagaṇitamū or Daśavidhagaṇitamū, the reasons for which are as follows: Mahāvīra's Sanskrit text consists of a Ṣaṭṣādhikāra and eight vyavahāras, namely 1. Parikarma, 2. Kalāsavarṇa, 3. Prakīrṇaka, 4. Trairāśika, 5. Miśraka, 6. Kṣetra, 7. Khāta and 8. Chāyā, to each of which a full chapter (thus 9 in all) is devoted. Mahāvīra enumerates these chapters at GSS I.20-23. Mallana translates these verses almost literally (p. 3, v. 9). This suggests that Mallana's work is also divided into (1+) 8 chapters.¹⁴ The same 8 titles are listed once again in a long prose passage at the end of the Ṣaṭṣādhikāra.¹⁵ But in certain mss. two more chapter titles are added to the list in this prose passage (9. Suvarṇagaṇitamū, and 10. Sūtragaṇitamū) and the whole passage bears the heading Daśagaṇitanirṇayamū.¹⁶ It is probable that some later author added these two extra chapters¹⁷ to the PG and made corresponding changes in the prose passage. Since then the work was also known under the name Daśagaṇitamū or Daśavidhagaṇitamū.¹⁸

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14. Mallana, however, employs the term *gaṇita* for *vyavahāra* and replaces Mahāvīra's *kalāsavarṇa* with *bhinna*.
 15. Pp. 14-15 of the printed edition to be described below. There is no parallel for this passage in the GSS.
 16. For instance, two mss. of the Government Oriental Manuscripts Library, Madras, have the heading Daśagaṇitanirṇayamū but enumerate only the first 8 titles within the prose passage; see A Triennial Catalogue of Manuscripts ..., vol. IV, part 3, Telugu, Madras 1934, R. no. 596, pp. 1508-1510; and A Descriptive Catalogue of Telugu Manuscripts ..., vol. X, no. 2282, pp. 2571-2574. The printed edition has the heading Granthakartr-pratiṣṭhā and the last two extra titles in parenthesis.
 17. Suvarṇagaṇita teaches calculations relating to the fineness of gold, but no other mathematical text devotes an exclusive chapter to this: cf. my "Varṇamālikā System of Determining the Fineness of Gold in Ancient and Medieval India" in: Aruṇa-Bhāratī, Professor A. N. Jani Felicitation Volume, Baroda 1983, pp. 369-89. From the extracts published in mss. catalogues, Sūtragaṇita appears to contain rules of conversion from one unit of measurement to another, from one era to another, and the like.
 18. Such designations of texts according to the number of chapters, sections, or verses, obtain in the case of Sanskrit texts as well, e.g. Āryabhaṭa's Daśagītikā or Śrīdhara's Triśatikā. In some mss., the GSS itself is styled Ṣaṭṣādhikā (see, Pingree, Census of the Exact Sciences in Sanskrit, A-4, p. 388), or Ṣaṭṣādhikā (GSS-J, appendix 5, p. 56), or Chattīsī (ibid., p. 55). Why it is called thus is explained in ms. no. 63 of the Balātkāra-gaṇa-mandira, Kārañjā, (see ibid., p. 55) in a mixture of Sanskrit, Prakrit and old Kannaḍa: *ahat-tisam etena sakala 8 bhinna 8 bhinnajāti 6 prakīrṇaka 10 trairāśika 4 imttā 36 nū chattīsam*, "Chattīsa: by this /ī is denoted/ 8 /fundamental

In spite of the great importance of the PG, there seems to be only one edition - that too incomplete - of this text:

Sārasaṅgrahaṅgamam, Pāvulūri Mallana (Mallikārjuna) praṅgamam. Ed. Veṅṅūri Prabhākara Śāstri. (Sri Venkateswara Oriental Research Series, no. 38. Editor, P. V. Ramanujaswamy). Pt. I. Tirupati 1952.¹⁹

This edition contains only the first two chapters: Samjñādhikāra and Parikarma are treated as the first chapter and given the title Parikarmagaṅgamam (pp. 1-91) and the Bhinnagaṅgamam (= Kalāsavarṇa-vyavahāra of the GSS) as the second (pp. 92-102).²⁰ But this second chapter stops with the 8 operations for simple fractions (= GSS III.1-53) and does not cover the six types of fractions (kalāsavarṇa-ṣaḍjāti, GSS III.54-140).

A description of the manuscript material used for this edition was promised for the second part which never appeared. All that we are told is that the edition is based on mss. (i) collected by S. P. L. Narasimhaswamy of Visakhapatnam, (ii) from the Government Oriental Manuscripts Library,

operations for/ whole /numbers/, /the same/ 8 /operations for/ fractions, 6 types of fractions, 10 /types of/ miscellaneous problems, 4 /varieties of/ the Rule of Three (i.e. Rule of Three, Five, Seven and Nine), these 36; /hence the name/ Chattīsa." Of course, these 36 items are covered in the first four vyavahāras only, but it is not clear whether this numerical designation refers to the first four vyavahāras only or to the entire text.

19. I am grateful to Professor P. Sriramamurti, Waltair, and Drs. D. Sridhara Babu and S. Sudarsana Sarma, Tirupati, for their help in obtaining a xerocopy of this edition.

P. T. Raju, Telugu Literature (Andhra Literature), Bombay 1943, p. 20, states: "Two mathematical works, Gaṅgita-sārasaṅgrahamam and Prakīrṇagaṅgamam, written by Pavuluri Mallana and Eluganti Peddana respectively, are now available." Cāgaṅṅi Śeṣayya, op. cit., pp. 200-207, also seems to be familiar with the full text of the PG. But all my efforts to locate such an edition proved futile. Nor is the General Editor of the Tirupati edition aware of any earlier edition except for 16 pages printed and then discontinued by his brother at Visakhapatnam (Preface, p. i). My search for the second work mentioned by P. T. Raju was equally unsuccessful. Cāgaṅṅi Śeṣayya, op. cit., p. 206, quotes two verses from this Prakīrṇagaṅgamam, where Peddana claims that his work is an improvement upon the PG.

20. In the first 35 pages, verses are not numbered. From p. 36 on, they bear separate serial numbers for each section. In my citations, the first number refers to the page and the second is the number of the verse in that particular page (up to p. 35) or the serial number of the verse in a particular section (beyond p. 35).

Madras, and (iii) from the Telugu Academy, Kakinada.²¹

The edition records several variant readings but does not attribute them to any specific manuscript. Without this information the value of the readings is much diminished and a future editor has to begin from the very beginning. Nor is the text available here quite satisfactory.²² For the present, however, we have to base our comments on the portion available in this printed edition consisting of the Samjñādhikāra, Parikarma and about one third of the Bhinna.

The PG is composed entirely in verse, though occasionally interspersed with short prose passages in the manner of the Telugu kāvyas. Mallana does not burden his language with long Sanskrit compounds like Nannaya, but writes in a simple idiomatic Telugu. We do not know of any other poetical work by him, even so the beautiful verses of the PG amply justify his claim of being "*sukavimalla*". That he can compose with great felicity in Sanskrit as well can be seen from two *maṅgala-śloka*s in praise of Śiva (1.1; 92.1). What is more important for a mathematical text is the following: in expressing numbers, he avoids, as far as possible, the *bhūtasamkhyā* or word-notation usually employed in Sanskrit texts and expresses the numbers as they are commonly spoken. Of course, for larger numbers, he does adopt the word-notation,²³ but here too his verses have a natural flow that is rare

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21. Preface, p. iv. It has not been possible to find out which mss. of these two libraries were utilised in this edition. Besides these two libraries, the Oriental Research Institute, Sri Venkateswara University, Tirupati, possesses 17 palm leaf mss. of the PG, as the Director kindly informs me.
22. For that matter, the Sanskrit text as available in GSS and faithfully reproduced in GSS-J is not completely satisfactory either. I give two instances of formal nature. Each chapter begins with a salutation to Jina, Vardhamāna or the Siddhas, but such a *maṅgala-śloka* is wanting at the beginning of the Parikarma-vyavahāra. Secondly, in the final stanzas of the first four *vyavahāras*, the name of the respective metre is cleverly incorporated. One would expect that the other four *vyavahāras* as well as the Samjñādhikāra would also end in the same manner, but that is not the case, (see also the next note). Incidentally, the four metres are *mattebhavīkrīḍita*, *utpalamālikā*, *pṛthvī* and *śālinī*: the first two are not usually met with in Sanskrit texts but are favourite metres in Telugu (and perhaps also in Kannaḍa).
23. GSS I.53-68 give a list of word-numerals for 1 to 9 and 0. These verses are found only in one ms. (M = from Mysore) and are not always metrically correct. The PG (p. 13) likewise supplies the word-numerals in a different set of Sanskrit verses which are also corrupt at places and the list does not contain all the terms used in the text. It is likely that both the sets are interpolations.

in Sanskrit texts with the possible exception of the Līlāvati.

A comparison of the portion available in the printed text with the Sanskrit original shows that the text of the PG contains a large number of verses for which there are no corresponding passages in the GSS. At this stage, it is difficult to say whether these additions are by Mallana himself or whether they are by some later writers. At the same time, however, the Telugu verses for which parallels exist in the GSS demonstrate that Mallana is quite faithful to the original, even with regard to the numerical quantities. For instance, this is Mahāvīra's first example under multiplication (GSS II.2):

*dattāny ekasmai jīnabhavanāyāmbujāni tāny aṣṭau /
vasatīnāṃ caturuttaracatvāriṃśacchatāya katī //*

Mallana renders it as follows (17.1)²⁴:

*melugan ēṇimidi yōkka śi-
vālayamuna kaṃbujamulan arpiṃcīnaco
nīlekka nūṭanaluvadi-
nālugu bhavanamula kēṇni nalinamul ayyēn.*

"Eight lotuses were offered to one Jina/Śiva temple. How many then are for 144 temples?" It should be noted that though there is a change of the deity - from Jina to Śiva - the numerical quantities (144 x 8) remain the same, and this is the case almost everywhere.

The following, however, is certainly Mallana's innovation. While Mahāvīra gives the units of measurement from what he calls Magadha (*māgadha-mānena*, GSS I.21), Mallana's metrology (pp. 4-11) consists of units then prevailing in the Telugu-speaking areas. These units are highly interesting for the history of the Telugu language and also for the economic history of the region.²⁵

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24. For the transliteration of Telugu (and later Prakrit) into Roman, I use the standard system used for Devanagari, with the addition of *ē* for short *e* and *ō* for short *o*.
25. The only study of Mallana's metrology that I am aware of is Vaidehi Krishnamurthy, Social and Economic Conditions in Eastern Deccan (1000 A.D. - 1250 A.D.), Madras 1970, p. 115, cited in: Saradha Srinivasan, Mensuration in Ancient India, Delhi 1975, pp. 55, 84, 88, 96, 113.

Otherwise, the additions pertain to problems or examples, but hardly to mathematical rules or methods of solving problems. In fact, the PG seems to reduce the number of methods to the minimum. While the GSS teaches 5 methods of squaring and 7 of cubing, the PG has only one each and avoids all algebraic solutions. In the printed text, additional problems occur in the sections on multiplication (45 extra) and division (21 extra). The main purpose of these problems is to achieve two peculiar formations of numbers.

(i) GSS II.10-15,17 contain problems on multiplication where the products consist of a symmetrical arrangement of digits like the beads or pearls in a necklace. Mahāvīra calls such products *kaṇṭhikā*, "necklace". For example, in II.10 we are asked to multiply 12345679 by 9 and obtain a *narapāla-kaṇṭhikābharāṇa*:

$$12345679 \times 9 = 111111111.$$

Or II.17:

11011011 \times 91 = 1002002001, where the multiplicand is also a "necklace" number.

Of the other mathematical texts in Sanskrit, only the Gaṇitatilaka of Śrīpati (p. 6, lines 8-9) has one such problem:

$$12987013 \times 77 = 100000001$$

and this product is described as *sphuṭa-tāraka-vartula-muktābharāṇam atra mahēśvarakaṇṭhe*. The commentary on Śrīpati's Siddhāntaśekhara XIII.2, while quoting GSS II.10 and 17, dismisses these two problems with the remark that they are meant for the amusement of children (*bālānaṃ vinodanārtham*).²⁶

Mallana, however, seems to delight in such children's amusements. Besides a faithful translation of GSS II.10-15,17, he gives several additional problems to achieve the "necklace" products.²⁷ The grandest of such products are the one containing 36 digits all of which are unities (31.1) and another of 22 digits where the first and the last are unities and the rest zeros (33.4).

26. Siddhāntaśekhara, ed. Babuaji Misra, vol. II, Calcutta 1947, p. 6.

27. Mallana uses the designation "Śiva's necklace", 30.1; 35.3.

(ii) A variation of this "necklace" number is where the digits go from 1 up to n in an ascending order and then continue in a descending order up to 1. Mallana has a special name for these numbers, "moon-like" (*hīmakaropama* 24.4), since here the digits first increase gradually and then decrease, like the phases of the moon in an *amānta* lunar month (which was and is current in South India). Thus he gives several pairs of multiplicands and multipliers to achieve 12345654321.²⁸ The same product is obtained in a more elegant way by multiplying six ones by six ones (24.4):

$$111111 \times 111111 = 12345654321.$$

If one carries out this multiplication on paper, it will be immediately apparent how one could have arrived at such magic numbers quite empirically and then looked for other sets of factors for them.

A variation of the "moon" product contains each digit twice (25.1):

$$124,703,828,407,532,110,986,407,282,703,579 \times 9 = 1122334455667788998877665544332211.$$

Other problems achieve the reverse of the "moon" products, i.e. the digits first decrease and then increase like the phases of the moon in the (northern) *pūrṇimānta* month, but a separate name is not given to these numbers. For instance, 26.3:

$$1,097,393,690,013,717,421 \times 9 = 9876543210123456789.$$

In the section on division also there are additional problems where the dividend is a "necklace" or "moon" number, and several of the problems here are the reverse of those on multiplication.

In the present state of our knowledge of the manuscript material, it is difficult to say whether these additional problems are Mallana's own or later interpolations. In either case, the fondness for such magic numbers shows the popularity of these number-games, especially so when these problems are clothed in mellifluous metres like *mattakokila*. The following is an example of this metre (42.22):

tarkasomulu darkabāhulu darkarāmulun abdhulun
darkasaṅkhyalu tarkasāyaka-tarkatarkavitānamul
tarkabhūdara-tarkadaṅtulu darkapadma-jārasulun
arkabhojuḍa śailarāmulan āra bālgōni cēppumā.

28. GSS II.7 achieves this product but without giving it a special name.

"Divide [successively] six (*tarka*) ones (*soma*), six twos (*bāhu*), six threes (*rāma*), fours (*abdhi*) in number six, six fives (*sāyaka*), six sixes (*tarka-vitāna*), six sevens (*bhūdhara*), six eights (*danti*), and six nines (*padmaja*) by 37 (*śaīla-rāma*, read from the right) and tell [the quotients]."

111111 : 37 = 3003; 222222 : 37 = 6006; ... 999999 : 37 = 27027.

There are seven such verses (41.19 - 43.25) with dividends ranging from three to nine digits.

The Triennial and Descriptive Catalogues of the Government Oriental Manuscripts Library, Madras, reproduce extracts from a large number of mss. containing the full text or single chapters of the PG. The extracts have often material that is clearly interpolated, though written in lucid verse, and successive generations of mathematicians seem to have added their own verses to the text of the PG, which originally may have consisted just of the translation of the GSS. This is a clear testimony to the fact that Mallana's Telugu rendering popularised mathematics in the Telugu country just as Nannaya's Telugu *Mahābhārata* made the Bhārata-myth almost a living reality to the Telugu people.

IV

The printed edition of the PG occasionally reproduces extracts from a *ṭīkā* without mentioning its author or the mss. from which these extracts are taken. Some mss. of the Government Oriental Manuscripts Library, Madras, contain a *ṭīkā*,²⁹ but these mss. too do not seem to mention the author. Even so, the few extracts available in the printed edition are interesting in several respects. For want of space, attention will be drawn here only to the Prakrit passages in the *ṭīkā*, which are valuable for the history of mathematical education in India. In the context of the rules for squaring, cubing and extracting square-roots and cube-roots, the *ṭīkā* cites the

29. Ms. nos. 2282, 2288, 2295 and 2296 in A Descriptive Catalogue of Telugu Manuscripts in the Government Oriental Manuscripts Library, Madras, vol. X, contain a *ṭīkā*. Of these, nos. 2282 and 2295 seem to contain the same *ṭīkā* but it is not certain whether this is the same as the one in the printed edition.

respective paradigms or tables in Prakrit. Since early specimens of arithmetical tables are not recorded, I reproduce below the Prakrit passages, exactly as they are printed.

(i) Table of squares (p. 46):

ěkkasa vargo ěkka
 biyyasa vargo cāri
 tiyyasa vargo navvā
 cāriśa vargo ṣoḷā
 paṃcasa vargo paṃu(ṣaṃcā)vīśā.³⁰
 chāyasa (chānnasa) vargo chatrīśā
 sattasa vargo navvetāḷā (navatāḷa)³¹
 aṭṭasa vargo cauṣaṣṭi
 navvasa vargo ekāśīti
 sunnasa vargo sunnā.

(ii) Table of square-roots (p. 49):

ěkkasa vargomūlo ěkkā
 cāriśa vargomūlo binni
 navvasa vargomūlo tinni
 ṣoḷasa vargomūlo cāri
 paṃnavīśa vargomūlo paṃca
 chattīśa vargomūlo cāhā
 navvetāḷasa vargomūlo satta
 cauṣaṣṭīśa vargomūlo aṭṭa
 ekāśītīśa vargomūlo navva
 sunnasa vargomūlo sunna.

(iii) Table of cubes (p. 53):

ěkkasa ghanno ěkkā
 biyyasa ghanno aṭṭā
 tiyyasa ghanno sattāvīśā
 cāriśa ghanno cauṣaṣṭi

30. It is not clear if the words in brackets here and below are variant readings from other mss. or emendations by the editor. In one case, at least, it seems to be an emendation: *navabinno(viṃśo)ttara*.

31. It is interesting to note that the *ekona-* or *īna-* form for the numerals 19, 29, ... 79 in north-Indian languages does not occur here. The forms *navvetāḷa* for 49 here and *navabinni* (or more correctly *navavīśa*) for 29 below are more akin to the Dravidian numerals.

pañcāsa ghanṇo paṇaviśottara-ekaśataṃ
 cāyasa ghanṇo ṣolottara-biṇṇiśataṃ
 sattasa ghanṇo tretāḷottara-tiṇṇiśataṃ
 aṭṭasa ghanṇo biṇṇiśottara(bārāhottara)-pañcaśataṃ
 navvasa ghanṇo navabinno(navaviṇṇṣo)ttara-sattaśataṃ
 sunṇasa ghanṇo sunṇā.

(iv) Table of cube-roots (p. 59):

ēkkasa ghanṇomūlo ēkkā
 aṭṭasa ghanṇomūlo biṇṇi
 sattaviśa ghanṇomūlo tiṇṇi
 cauṣaṣṭiśa ghanṇomūlo cāri
 paṇaviśottara-ekaśata ghanṇomūlo pañcā
 soḷāhottara-biṇṇiśata ghanṇomūlo cāhā
 tretāḷottara-tiṇṇiśata ghanṇomūlo sattā
 biṇṇiśottara-pañcaśata ghanṇomūlo aṭṭā
 navabinno(viṇṇṣo)ttara-saptaśata ghanṇomūlo navvā
 sunṇasa ghanṇomūlo sunṇa.

Unfortunately, there are no full multiplication tables in these extracts but just a few fragments. While explaining in Telugu how to perform certain mathematical operations step by step, the *ṭikā* cites single lines from the four tables given above and also from multiplication tables in the following manner: "Since the square-root of nine is three (*navvasa vargomūlo tiṇṇi*), write down three," or "since six sixes are thirtysix (*cha chakkaṃ chātrīsā*), put down thirtysix," and so on. I give below the fragments of the multiplication tables.

p. 50: *cha chakkaṃ chātrīsā*
 54: *cāri tiyyaṃ bārā*
 60: *sapta tiyyaṃ yakkāvīsā*
 61: *bi tiyyā cāhā*
 tiṇṇi tiyyā navvā
 bārā tiyyā chātrīsā.

There is no doubt that all these are mnemonic tables taught to children. The peculiar phonology with elongated vowels (e.g. *vargomūlo*, *ghanṇomūlo* etc.) suitable for recitation in a singsong fashion and the fact that the *ṭikā* quotes from these tables in the middle of Telugu passages, thus anticipating the knowledge of these tables, indicate that at some point

after the 11th century somewhere in the Telugu country children learnt by rote these Prakrit tables.

It is, of course, well known that, unlike school children today who memorise only the multiplication tables - that too up to 10 only -, children in earlier ages had to learn many other arithmetical paradigms by heart,³² but this is the earliest evidence to show that tables of squares, square-roots, cubes and cube-roots were taught in addition to the common multiplication tables. The line *bārā tiyyā chatrīsā* (12 x 3 = 36) suggests that the multiplication tables ran beyond 10.

The modern Telugu word for multiplication tables is *ēkkālu* in plural (singular *ēkkamu* or *ēkkaṁ*) for which no satisfactory etymology seems to be available so far. In the *ṭikā*, the four tables are preceded by a line each: *vargamūlapu magga*, "magga for square-root" (p. 45); *ghanamūlapu magga*, "magga for cube-root" (p. 59); *iṇḍuku magga*, "magga for this," i.e. square (p. 45), cube (p. 52). *Magga*, then, was an earlier Telugu term for arithmetical table (from Sanskrit *mārga*, "path, paradigm (?)"). Interestingly, the Kannada word for multiplication table is *maggi*.³³ But where does *ēkkālu* come from?

On the basis of the fragments given above, we can reconstruct the multiplication table for One, which may have read thus:

ēkkā ēkkaṁ ēkkā /
ēkkā biyyaṁ binni /
ēkkā tiyyaṁ tinni /
ēkkā caukaṁ cāri /
ēkkā paṇcaṁ paṇcā /
ēkkā chakkaṁ cāhā /...

This reconstruction immediately suggests that the Telugu term *ēkkālu* is nothing but the initial word of the Table of One plus the plural particle

32. At the beginning of this century, Gujarati Bania boys had to commit to memory multiplication tables for whole numbers and fractions, interest tables, tables of squares and so on; cf. Gazetteer of the Bombay Presidency, IX.1, Bombay 1901, p. 80.

33. F. Kittel, A Kannada-English Dictionary, Mangalore 1894, p. 1184, s.v. *maggi*.

(*ġkkā + lu*). If this etymology is correct - the German expression "Einmal-eins" for multiplication table is made up of the first two words of the Table of One in an analogous manner -, these Prakrit tables must have been in use in the Telugu country rather extensively to give rise to a new term *ġkkālu*, and this term must have designated originally all arithmetical paradigms, beginning with the multiplication tables.

But this raises another problem. In the eleventh century, Mallana felt it necessary to translate the GSS into Telugu, but the *ṭīkā* written in a subsequent period suggests the use of the Prakrit tables. It is true that the earliest literary records of the Telugu country are the inscriptions in Prakrit, but Prakrit ceased to be employed in inscriptions here around the second half of the 4th century A.D.,³⁴ and by the 11th century it was neither a popular nor an official medium. Does this mean that when Mallana composed the PG and when the anonymous *ṭīkā* was written still later, there were no Telugu tables available and that children had to mug up these tables in Prakrit?

If this was so, can we then assume that the following stages took place in the use and spread of these tables? (i) The name *magga*, which survives still in Karṇāṭaka, suggests a Karṇāṭaka origin of the Prakrit tables, presumably among the Jainas. (ii) From Karṇāṭaka, the tables spread to the Telugu region and were popular there under the name *magga* when the *ṭīkā* was written some time after the 11th century. (iii) In the Telugu region, some time after the *ṭīkā* was written, they gave rise to a new name *ġkkālu*, which was probably a children's nickname. (iv) At a subsequent period, they were replaced by tables in Telugu, but the name *ġkkālu* was retained.

This article, it will be noticed, ends in questions rather than answers. But then that is perhaps the appropriate attitude in a paper presented to one's Guru from whom one continues to receive much inspiration, affection and generous help in one's academic ventures.

34. D. C. Sircar, Indian Epigraphy, Delhi 1965, p. 43ff.