SRI VENKATESWARA UNIVERSITY

ORIENTAL JOURNAL

VOL. XXVIII JANUARY - DECEMBER, 1985 PARTS 1 & 2

SREERAMULA RAJESWARA SARMA

MATHEMATICAL LITERATURE IN TELUGU: AN OVERVIEW *

The *Pāvulūrigaṇitamu* (=PG), a Telugu rendering of Mahāvīra's *Gaṇitasārasaṃgraha*[1] (=GSS) by Pāvulūri Mallana, is generally accepted as a product of the late 11th century. Mallana states that Rājarāja gave an *agrahāra* to his grandfather, also called Mallana. Hence the grandson Mallana must be a younger contemporary of Nannaya and the PG the second extant work in Telugu after Nannaya's *Āndhramahābhāratamu*. At the same time, the PG is also the earliest known translation of a scientific text from Sanskrit into any regional language.

Thus the importance of PG, whether in the history of Telugu literature, or in the history of mathematics in the Andhra region, or in the history of translations of scientific texts in India, can hardly be overemphasised.[2] Yet so far not much interest has been shown in the study of PG.[3] In fact, the full text is not even available in print. Only a small portion was published from Tirupati in 1952 and this was edited by the great Telugu savant Veturi Prabhakara Sastri.[4] About the history of mathematical literature after Mallana we know still less.[5] We are thus depriving ourselves of a part of our heritage which ought to be as precious as any other aspect of Telugu past. In this paper I wish to present the little we know about the history of mathematical literature in Telugu and plead for systematic efforts to study this sadly neglected area.

As translator, Mallana's performance is impressive. The lucidity with which he renders the terse Sanskrit of Mahāvīra is worth emulating by every modern translator of scientific texts. His way of handling mathematical rules or examples containing large numbers — some examples have as many as 36 digits — is unrivalled even in Sanskrit. But is Mallana just a good translator, or did he make any original contribution of his own? This cannot be answered either way until there is a good critical edition of the full text or a complete picture of the manuscript tradition of PG.

Nevertheless, on the basis of the printed edition from Tirupati and the descriptions of the mss of PG at the Government Oriental Manuscripts Library, Madras,[6] (=GOML) some tentative conclusions can be drawn. The original GSS in Sanskrit contains a *saṃjñādhikāra* and eight chapters on so many topics: 1. *parikarma*, 2. *kalāsavarṇa*, 3. *prakīrṇaka*, 4. *trairāśika*, 5. *miśraka*, 6. *kṣetra*, 7. *khāta* and 8. *chāyā*. The Tirupati edition of PG breaks off in the middle of the second chapter. A comparison of this edition with the Sanskrit original shows that PG omits certain portions but adds many others. Thus, while GSS teaches 5 methods of squaring and 7 of cubing, the Telugu version has only one each and avoids all algebraic methods. At this stage it is difficult to say whether this omission is due to Mallana himself or to a later redactor. The additions, on the other hand, are of immense interest to us.

The *Saṃjñādhikāra* of GSS defines the numerical and metrological terminology. Of numbers, it enumerates the names of 24 decimal notational places, that is from 10^0 to 10^{23}. Instead of translating these Sanskrit stanzas, PG (p.12) just reproduces them but adds three more stanzas in Sanskrit, thus continuing the notational places up to 36.[7]

In the Sanskrit original, Mahāvīra gives the pan-Indian units of measurement, which he calls *magadhamāna*. In their stead, Mallana offers the units prevailing in Āndhradeśa in his time. It is fairly certain that these are not interpolations but were introduced by Mallana himself, since the same units are employed in several examples. These units pertain to the following measurements : 1. *bhūmipramāṇamu* (linear measure), 2. *kuṃṭa*⁰ (area), 3. *udaka*⁰ (volume of liquids), 4. *dhānya*⁰ (volume of grain), 5. *kāṃcana*⁰ (weights of gold), 6. *tulā*⁰ (weights of other commodities), and 7. *kāla*⁰ (time) (pp. 4-11). It is needless to emphasise that the names of these units are of great value both for the economic history and the history of Telugu language.

The third type of addition relates to mathematics proper. PG contains some 45 additional examples under multiplication and 21 under division, which are not found in Sanskrit. All these extra examples have one common feature, viz, to produce numbers containing a symmetric arrangement of digits. The Sanskrit original has also a few and Mahāvīra calls them "necklace numbers" (*kaṇṭhikā*) because the symmetric arrangement of digits is like the symmetric arrangement of beads in a necklace.[8] PG abounds in necklace of diverse patterns. For example, necklaces made up of just unities :

$$111 = 37 \times 3$$

$$1111 = 101 \times 11$$

$$11111 = 271 \times 41$$

....

$$111111111 = 37 \times 3003003 = 3 \times 37037037$$

$$111111111111 = 37 \times 3003003003003 = 101 \times 1100110011$$

and finally 111 111 111 111 111 111 111 111 111 111 111 111 =

1 763 668 430 335 097 001 763 668 430 335 097 x 63,

or with unities intermingled with pearl-like zeros

100010001 = 14287143 x 7

1100110011 = 157158573 x 7

1000000001 = 142857143 x 7 = 1298013 x 77

10101010101 = 3367003367 x 3 = 777000777 x 13.

And here we have the largest pearl necklace :

10 000 000 000 000 000 000 01 =20 408 163 265 306 122 449 x 49

Mallana introduces a new formation and calls it "moon-like" number because here the digits increase from 1 to *n* and then decrease up to 1 just as the phases of the Moon gradulally increase and then decrease in an *amānta* lunar month,e.g.

12345654321 = 111111 x 111111 (p. 24[9]).

A variation of this is where each digit occurs twice as in

1122334455667788998877665544332211 =

124 703 878 407 532 110 986 407 282 703 579 x 9.

There are also reverse cases of the Moon numbers with digits first decreasing from *n* upto zero and then increasing upto *n*, like the Moon's phases in a *pūrṇimānta* month :

6543210123456 = 146 053 847 x 448

9876543210123456789 = 1 097 393 690 013 717 421 x 9 (p. 26[10])

and other amusing formations without a label like

777778888899999 = 2 710 030 971 777 x 287

112233445566778899 = 12 470 382 840 753 211 x 9

9 x 6666666655555555 = 5 9999999 8 9999999 5.

It should be noted that often for one product several sets of factors are given. Again it is difficult to say whether all these are Mallana's own innovations or interpolations of a later age. But even if they are interpolations, they show the fondness of the Telugus for such magic numbers, and this predilection may have been engendered by Mallana through his mellifluous verses like the following in *Mattakokila* metre :

aṣṭacaṃdrulun aṣṭabāhulun aṣṭarāmulun aṃbudhul
aṣṭasaṃkhyalun aṣṭabāṇamul aṣṭaśāstravitānamul
aṣṭaśailamul aṣṭahastulun aṣṭanaṃdanarāsulun
sṛṣṭilopala nūṭayŏkkaṭa jĕppu pālgona labdhamul.
(p. 42).

The verse instructs us to perform the following divisions:

11111111 ÷ 101 = 1100011; 22222222 ÷ 101 = 2200022
33333333 ÷ 101 = 3300033;....99999999 ÷ 101 = 9900099.

Note that here all the three quantities - the dividend, divisor and quotient - are necklace numbers.

It is indeed likely that problems such as these which produce startling results attracted the attention not just of serious mathematicians who invented more problems like these but also of laymen who posed these problems as puzzles or riddles under the village tree.

Reverting to PG, while it is difficult to say whether Mallana added anything new into the chapters, it is certain that his Telugu version contained (1+) 8 chapters like the original. At a later stage, two new chapters called *Suvarṇagaṇitamu* and *Sūtragaṇitamu* were added and PG came to be known as the *Daśavidhagaṇitamu*.

From the description of the mss of PG at GOML, one gets the impression that the popularity of PG, paradoxically enough, inhibited the composition of new mathematical texts in Telugu. Instead of writing a new book, successive generations of mathematicians added their contributions, often in equally beautiful verse, into the appropriate chapter of PG. Only a few of these can be identified as interpolations as they refer to persons or events posterior to Mallana.[11] Copyists also interpolated verses from other sources. Furthermore, as with most of such popular books, mss containing single chapters began to be made and these continued a separate existene of growth and interpolation.[12]

The chapter on area measure called *Kṣetragaṇitamu* has been the *vade-mecum* of the village *Karaṇams* through the centuries and in their hands it underwent changes of all kinds; in particular, units of length and area were changed in accordance with the local usage. At the beginning of the last century, one such copy fell into the hands of Benjamin Heyne who translated it with the help of "a Brahmin above eighty years of age ... who could repeat the greatest part of the work by heart."[13] The text Heyne used went through so many changes that it is hardly recognizable as a translation of GSS. Therefore, the reconstruction of PG as Mallana may have written it is going to be a challenging task.

According to two stanzas quoted by Cagamti Sesayya, one Elugamti Peddana was not satisfied with the way Mallana rendered the second and third chapters of GSS into Telugu and so wrote his own *Bhinnaprakīrṇagaṇita*.[14] P. T. Raju says: "Two mathematical works of the eleventh century, *Gaṇitasārasaṃgrahamu* and *Prakīrṇagaṇitamu*, written by Pāvulūri Mallana and Elugaṃṭi Peddana respectively, are now available."[15] But my search for a printed edition or a ms of Peddana's work has been in vain, nor do I know on what evidence P. T. Raju assigns Peddana also to the 11th century. From the above-mentioned stanzas it appears that by Peddana's time, PG was already known as *Daśavidhagaṇitamu*.

The rest of the known history of mathematical literature in Telugu can be related in a few sentences. In the 16th century, Koḍūri Vallabha or Vallabharāya, son of Rāghavasūri and grandson of Erraya, translated Bhāskara's *Līlāvatī* into Telugu at the instance of Bommalāṭa Kāḷayya, an officer at the court of Acyutarāya of Vijayanagar.[16] At the end of the 16th or the beginning of the 17th century, Piṃgaḷi Veṃkaṭādri, a descendant of Piṃgaḷi Sūrana, wrote *Kṣetragaṇitamu* based on PG.[17] In the last quarter of the 19th century, Tadakamalla Veṃkaṭakṛṣṇārāvu translated Bhāskara's *Līlāvatī* and *Bījagaṇita* into Telugu.[18] GOML has a few other anonymous texts, one of which has the title *Veṃkaṭeśagaṇitamu* because it is dedicated to the deity of that name.[19] This is all I could gather from the catalogues of GOML. But there are many other manuscript collections and there must be many more mss in private hands. I came across two mss through a chance enquiry and I have hopes that an organised effort will bring many more mathematical texts to light.

As I was looking for mss of PG during my last visit to Andhra Pradesh, Sri Mantri Gopala Krishna Murti, a paternal friend, very generously gave me two copybooks filled by his father, Mantri Panakalu Rayudu (1883-1928). The latter was a school master in many villages in the then Guntur district. His main passion, however, was mathematics and he was fond of asking deceptively simple mathematical riddles. He read widely and collected material on mathematics from both Telugu and Sanskrit sources in his copybooks. From him, his son inherited remarkable abilities of traditional computation: with amazing speed he once constructed for me a magic square of the order twentyseven. Of the many copybooks filled by his father, Gopala Krishna Murti could rescue only two; but these contain much interesting material. Besides a large extract of PG, there is a small treatise in 40 stanzas covering the whole range of arithmetic. There is no title, nor author's name, but it must emanate from Muktinutalapadu near Ongole, since each stanza concludes with the refrain *muktinūtalapāṭi-sthalakhelā dīnabandho veṇugopālakā*.[20] There are also some stray verses from a *Muktinūtalapāṭi-*

Nṛsiṃhajña-Gaṇitamu, invoking this time *callamalleśa, muktipuranivāsa, īśā*. The mss also contain an anonymous chapter on *trairāśika*, and some mathematical riddles in Telugu and Sanskrit.

But the most remarkable find is the solution to what is known as Josephus problem. The problem consists in arranging in a circle two equal groups of good persons and bad fellows in such a manner that each n-th person to be removed from the circle must be a bad fellow. Though named after the Jewish historian Flavius Josephus (37-100 A.D.), this problem was not known in Europe before the 10th century. Japan is the only other place where this problem was known. It became popular there some time after the 12th century.[21] This is the first time that I come across this problem in any Indian source. The Telugu version which I now discovered runs as follows: 15 brahmins and 15 thieves had to spend a dark night at an isolated temple of Durgā. At midnight, the goddess appeared in person and wanted to devour just 15 persons because she was hungry. The thieves naturally suggested that she should consume the 15 soft-limbed brahimins. But the brahmins proposed that all the 30 would stand in a circle and that Durga should eat each ninth person. The proposal was accepted by Durga and the thieves. So the brahmins arranged themselves and the thieves in a circle, telling each one where to stand. Durga counted out each ninth person and devoured him. When 15 were thus eaten, she was satiated and disappeared, and only brahmins now remained in the circle. The problem is: how to arrange the brahmins and thieves in the circle.[22] The answer is couched in a *Mattebhavikrīḍita*:

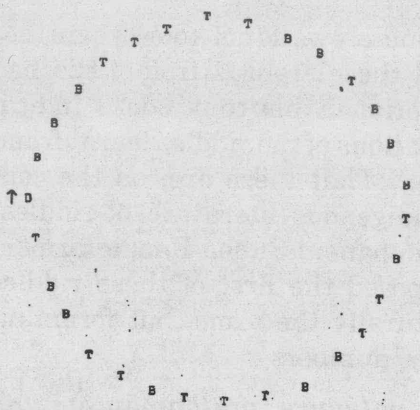
yuga-bāṇākṣi-dharāgni-candra-śaśi-bāhu-dvandva-rāmeṃdulaṃ

yugaḷī-karṇakalā-dvisaṃkhya-guḍigān ūhiṃci yantyastha vi-

pragaṇaṃ bādiga dŏmmiḍiṃṭa kramam ŏppaṃ durga bhakṣiṃpuco

mŏgi viprul jayam ŏṃdi taṣkarulu nirmūlambulair aṃdarun.

Here we have the alternating numbers of brahmins and thieves to be positioned in the circle: 4 brahmins, 5 thieves; 2,1; 3,1; 1,2; 2,3; 1,2; 2,1.



Arrangement of 15 brahmins (B) and 15 thieves (T) in a circle. The starting point is indicated by the arrow.

Another variant - it was not known in Europe - is so to arrange 30 brahmins and 30 thieves that each 12th person to be removed must be a thief. The solution is given in a *Campakamālikā* :

aruvadi yaṃkĕlum ravulan aṃdina corulu nilva viprulau -

duru dharaṇīṃdu-candra-kara-dor-bhuja-bhū-guṇa-vahni-bhūvi-bhū-

dharaṇi-dharākṣi-soma-vasudhā-kṣiti-bhūrvi-dharitri-vārdhi-doḥ-

sthiralu śaśi-dvi-tarka-śara-cid-dharaṇīṃdu-dṛg-abdhi-somulun.

Here also there are pairs of numbers, the first one of brahmins and the second of thieves : *dharaṇi, iṃdu* (1, 1,) ; *candra, kara* (1, 2); *dor, bhuja* (2, 2); *bhū, guṇa* (1, 3); *vahni, bhū* (3, 1); *urvi, bhū* (1, 1), *dharaṇi, dhara* (1, 1); *akṣi, soma* (2, 1); *vasudha, kṣiti* (1, 1); *bhū, urvi* (1, 1); *dharitri, vārdhi* (1, 4); *doḥ, sthira* (2, 1); *śaśi, dvi* (1, 2); *tarka, śara* (6, 5); *cit, dharaṇi* (1, 1); *iṃdu, dṛg* (1, 2); *abdhi, soma* (4, 1).

Unfortunately we do not know where the late Pānakālu Rāyuḍu copied these stanzas from. His background and the other material in his copy books indicate that these are not versifications of the riddles learnt from some modern western source. That these are, on the contrary, part of the floating indigenous literature of riddles is confirmed when Sri Nannapaneni Subba Rao, a farmer in my village near Ongole, posed the first of these riddles to me. His solution is naturally the same, but formulated differenty, using ordinary numbers :

viprulu naluvuru, prathamāṃśamunan aidu (4, 5)
toṭa viprulu rĕṃdu, dŏṃga yŏkaḍu (2, 1)
dvijulu mugguru, sthiramugān ŏkka dŏṃga (3, 1)
dharaṇīśvaruḍ okaṃḍu, daskarulu reṃḍu (1, 2)
viprul iddaru, viḍi doṃgalu mugguru (2, 3)
brāhmaṇuḍ okkaḍu, baṃṭulu reṃḍu (1, 2)
agrajanmulu rĕṃḍu, aṭumīdan ŏka dŏṃga (2, 1).

Two versions of the same riddle in the same geographic area does indeed demonstrate the wide popularity of mathematical riddles in Andhra Pradesh[23]. Whether this is an offshoot of the popularity of the mathematical literature, or whether riddles — mathematical or otherwise – are transmitted in a different process independent of literature, is a question I am not competent to answer. But a collection of such mathematical riddles would certainly enrich the history of mathematics.

I may conclude this paper with a plea that an organised effort should be made by some university of Andhra Pradesh to save this mathematical heritage of the Telugus. The effort should be directed first at a survey and collection of all the mss on mathematics in public and private collections, and then at preparing critical editions, at least of the *Pāvulūrigaṇitamu*.

Notes and References

* Read at the section on Telugu Language, Literature and Culture, All India Oriental Conference, 34th session, Visakhapatnam, January 1989.

1. Ed. & tr. by M. Rangacharya, Madras 1912.
2. See my "The Pāvulūrigaṇitamu: the first Telugu Work on Mathematics," *Studien zur Indologie und Iranistik*, 13-4 (1987), 163-176.
3. It is gratifying that this paper apparently inspired one member of the audience at Visakhapatnam to read PG, cf. K. Kusumabai, 'Sārasaṃgrahagaṇitamu',: *Telugu, Vaijñānika Māsapatrika*, 3.3. (March 1989) 92-97.
4. *Sārasaṃgrahagaṇitamu, Pāvulūri Mallana (Mallikārjuna) praṇītamu*, ed. Veṭūri Prabhākara Śāstri, part 1: *Parikarma-bhinna-gaṇitamulu*, Tirupati 1952 (Sri Venkateswara Oriental Series, No. 38).
5. All that we have are two small essays: K. R. Rajagopalan, 'Mathematics in Andhra,' *Bhavan's Journal*, 6.8 (November 1959) 47-49; R. C. Gupta, "Some Telugu Authors and Works on Ancient Indian Mathematics" in: *The Souvenir of the 44th Conference of the Indian Mathematical Society*, Hyderabad 1978, pp. 25-28.

6. *Cf. A Descriptive Catalogue of the Telugu Manuscripts in the Government Oriental Manuscripts Library*, Madras, vol. X, Madras 1949, Nos. 2282-95.

7. The terms given by Mahāvīra are : 1. *eka* (10^{0}), 2. *daśa* (10^{1}), 3. *śata* (10^{2}), 4. *sahasra* (10^{3}), 5. *daśasahasra* (10^{4}), 6. *lakṣa* (10^{5}), 7. *daśalakṣa* (10^{6}), 8. *koṭi* (10^{7}), 9. *daśakoṭi* (10^{8}), 10. *śatakoṭi* (10^{9}), 11. *arbuda* (10^{10}), 12. *nyarbuda* (10^{11}), 13. *kharva* (10^{12}), 14. *mahākharva* (10^{13}), 15. *padma* (10^{14}), 16. *mahāpadma* (10^{15}), 17. *kṣoṇī* (10^{16}), 18. *mahākṣoṇī* (10^{17}), 19. *śaṅkha* (10^{18}), 20. *mahāsaṅkha* (10^{19}), 21. *kṣiti* (10^{20}), 22. *mahākṣiti* (10^{21}), 23. *kṣobha* (10^{22}), 24. *mahākṣobha* (10^{23}).

To these Mallana adds 12 more terms : 25. *nidhi* (10^{24}), 26. *mahānidhi* (10^{25}), 27. *parata* (10^{26}), 28. *ananta* (10^{27}), 29. *bhūri* (10^{28}), 30. *mahābhūri* (10^{29}), 31. *meru* (10^{30}), 32. *mahāmeru* (10^{31}), 33. *bahuśa* (10^{32}), 34. *bāhuśa* (10^{33}), 35. *samudra* (10^{34}), 36. *sāgara* (10^{35}).

8. GSS, *Parikarmavyavahāra*: *narapāla-kaṇṭhikābharaṇa* (*v.* 10), *kaṇṭhābharaṇa* (*vv.* 11, 15,), *ratnakaṇṭhikābharaṇa* (12), *rājakaṇṭhikābharaṇa* (13), *kaṇṭhikā rājaputrasya yogyā* (17).

9. *ārun ŏkkaṭlan ŏḍḍugān amarabĕṭṭi*
yaṃtaguṇakaṃbu cetanun amarabeṃci
sŏridi vargiṃci janulaku jodyamuganu
himakaropama-labdhaṃbun enayavaccu.

10. *śaśi-karābdhi-śaila-candra-hayāgnīṃdu-*
gagana-diva-nidhāna-karma-rāma-
nava-guṇādri-raṃdhra-diva-bhūmi-mita-rāśi
grahaguṇaṃbu seyu gaṇaka-tilaka.

11. *A Descriptive Catalogue* ... (see n. 6 above), No. 2294, where Pratāparudra is mentioned.

12. Ibid. Nos. 2296-2303 are independent mss of the *Sūtragaṇita*.

13. *Tracts, Historical and Statistical, on India, with Journals of Several Tours through various parts of the Peninsula*, London 1814, pp. 172-180: "A Free Translation of the Chetri [*kṣetra*] Ganitam, or Field Measuring of the Hindoos".

14. Cāgaṃṭi Śeṣayya, *Āndhra Kavi Taraṃgiṇi*, vol.1, second ed., Kapileśvarapuramu 1955, p. 206:
ghanuḍai vīrācāryulu
tana mahiman vĕlayajesĕ daśavidhagaṇitā -
lanu paddhatulu dharitrini
munupe yaṣṭādhikāra–mulu modalaniyun.
vīrācāryulu saṃskṛtāna gaṇitāl vikhyātigā jesĕ dān ā
rūḍhiṃ badi pāvulūri kavimalluṃḍ addi telgiṃcĕ nan
bere gāni guṇiṃpaleru gaṇakul bhinnaprakīrṇaṃbul
tīrainann elugaṃṭi peddana yavin dĕlpĕn guṇiṃpan
dharan.

15. *Telugu Literature,* Bombay 1943, p. 20.

16. A *Descriptive Catalogue* ..., No. 2280.

17. Ibid. No. 2275.

18. Ibid. Nos. 2274, 2279.

19. Ibid. No.2281.

20. I hope to publish this text soon with a mathematical commentary. Given below is a specimen, which states that the area of a circle, when multiplied by 14 and divided by 11, is equal to the square of its diameter and that the area divided by a quarter of the diameter equals the circumference (c).

ilamānaṃbu caturdaśeśvarulace hĕcciṃci bhāgiṃci yā
phalamuṃ mūlamu śeya vyāsam agu tatpādāṃśace dhāruṇī
phala-rāśiṃ bhajiyiṃpa sthūlavalayaṃbau vṛttabhū muktinū -
talapāṭi-sthalakhelā dīnajanabandho veṇugopālakā.

since $a = \pi d^2/4$, $d = \sqrt{a x 14} \div 11$

since $c = \pi d$, $a \div d/4 = \pi d^2/4 x 4/d = \pi d = c$.

21. Joseph Needham, *Science and Civilisation in China*, Vol.3, Cambridge 1959, pp.61-62; James R. Newman, *The World of Mathematics*, vol.4, second ed., London 1961, pp. 2428-2429.

22. In Germany, the solution is provided by the mnemonic line "Gott schl*u*g d*e*n m*a*nn *i*n *A*m*a*l*e*k, d*e*r *I*sr*ea*l b*e*zw*a*ng, " where the numerical values of the successive vowels ($a = 1$, $e = 2$, $i = 3$, $o = 4$, $u = 5$) indicate alternately the numbers of the good men and bad fellows to be arranged in the circle.

23. After this paper was completed, I came to know that this problem is discussed in the *Peddabālaśikṣa*. Perhaps it is the sourse of dissemination.

ISSN 0081 - 3907

1985